Launch *Eureka Math*: *A Story of Ratios®* *A Story of Functions®*

Virtual Engagement Materials

Table of Contents

Pre-Engagement Materials - Complete the following pages and come prepared with your notes and thoughts to have meaningful collaborative conversations throughout this virtual session. Directions for each task are listed at the top of the relevant page.

Session 1 Materials

Interim Materials - Read and annotate the following page. Come prepared with your notes and thoughts to have meaningful collaborative conversations throughout this virtual session. Directions for each task are listed at the top of the relevant page.

Session 2 Materials

Post-Engagement Materials

Launch *Eureka Math*

Three Key Shifts—Focus, Coherence, Rigor

Directions

Directions: While reading, annotate and consider your response to the following Guiding Question:

1. What one statement, from each Key Shift, resonates with you the most? Why?

Focus: Greater focus on fewer topics

The Common Core State Standards for Mathematics[1] ask math teachers to narrow the scope of content and deepen the focus on topics they cover in the classroom. As a result, the time and energy spent in the classroom changes from rapid movement through many topics to deeper focus on the work most important to achieving the Standards in the grade band. This focus helps students achieve at higher levels because they gain "a solid understanding of concepts, a high degree of procedural skill and fluency, and the ability to apply the math to solve problems in and outside the classroom" at ever-greater levels of complexity (NGA Center and CCSSO, n.d.).

Coherence: Linking topics and thinking across grades

Math makes sense. Coherence demonstrates to students that knowledge of a few principles leads to reasoning about any number of mathematical concepts and expressions. Coherence is all about the logical connections between topics and between grades, an area in which *Eureka Math*™ excels. The Standards develop principles within grades and in progressions from grade to grade, connecting learning so students build new understanding from prior knowledge. Each standard, therefore, is an extension of previous learning and foundational to future learning.

Rigor: Pursue conceptual understanding, procedural skills and fluency, and application with equal intensity

"Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades." To achieve this command of concepts, the Standards set explicit expectations for conceptual understanding, procedural skills and fluency, and application, the three aspects of rigor. To help students meet the Standards, the work in each grade must be balanced in approaching these three aspects of rigor, and teachers must pursue all three "with equal intensity" (NGA Center and CCSSO, n.d.).

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Coherence from Kindergarten through Precalculus and Advanced Topics

Directions
Analyze the following expressions, and consider: What might be the value of writing or saying the unit as part of the expression?

5 cars + 3 cars

5 tens + 3 tens

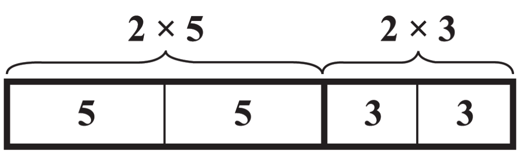
5 meters + 3 meters

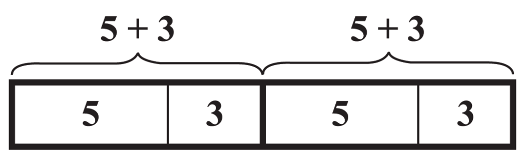
5 fours + 3 fours

5 ninths + 3 ninths

$5x + 3x = 8x$

Use the tape diagrams to answer the following questions.





How many fives are in the model? _____

How many threes are in the model? _____

How many fives are in the model? _____

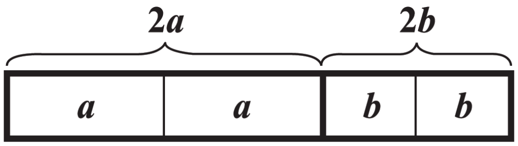
How many threes are in the model? _____

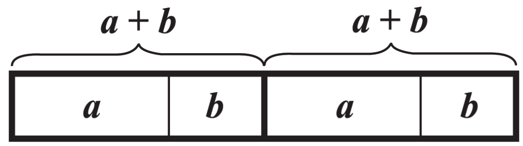
Write at least one expression to represent each model.

Write down ways of representing each expression with words.

Grade 6

a. Use the model to answer the following questions.





How many a's are in the model? _____ How many a's are in the model? _____

How many b's are in the model? _____ How many b's are in the model? _____

Write at least one expression to represent each model.

b. Model the double of $(5x + 3y)$. Are there terms we can combine?

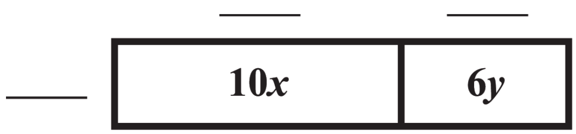
What is an equivalent expression that we can use to represent the double of $(5x + 3y)$?

Grade 7

a. Model $2(5 + 3)$ as a rectangular array.

b. Model $2(5x + 3y)$ as a rectangular array. Let the variables x and y stand for positive integers.

c. Let the variables x and y stand for positive integers, and let $10x$ and $6y$ represent the area of the two regions in the array in square units. Determine the length and width of each rectangle if the width is the same for each rectangle.



Algebra I

a. In *A Story of Units*, students compute 15×13 as follows:

b. Use the tabular method to multiply $(x + 5)(x + 3)$.

c. Use the reverse tabular method to factor $x^2 + 2x - 15$.

Algebra II

Use the reverse tabular method to calculate $(x^3 + 8x^2 + 20x + 25) \div (x + 5)$.

Write down some aha moments you had during this study of coherence.

What new understandings did you gain?

How might the Kindergarten through Precalculus and Advanced Topics coherence of the expression benefit your students?

Module Overview Note Catcher

Directions

1. Write a description of the important information in the Module Overview item.
2. Note whether the Module Overview item provides information about the coherence within the grade level and/or across grade levels. You might use a check or a brief descriptor.

Item	Description	Coherence	
		Within Grade	Across Grades
Table of Contents			
Overview			
Focus Standards			
Foundational Standards			

Item	Description	Coherence	
		Within Grade	Across Grades
Focus Standards for Mathematical Practice			
Terminology			
Suggested Tools and Representations			
Assessment Summary			

Module Overview Excerpt

Table of Contents[1]

Ratios and Proportional Relationships

[1]Each lesson is ONE day, and ONE day is considered a 45-minute period.

Grade 7 • Module 1

Ratios and Proportional Relationships

OVERVIEW

In Module 1, students build upon their Grade 6 reasoning about ratios, rates, and unit rates (**6.RP.A.1**, **6.RP.A.2**, **6.RP.A.3**) to formally define proportional relationships and the constant of proportionality (**7.RP.A.2**). In Topic A, students examine situations carefully to determine if they are describing a proportional relationship. Their analysis is applied to relationships given in tables, graphs, and verbal descriptions (**7.RP.A.2a**).

In Topic B, students learn that the unit rate of a collection of equivalent ratios is called the *constant of proportionality* and can be used to represent proportional relationships with equations of the form $y = kx$, where k is the constant of proportionality (**7.RP.A.2b**, **7.RP.A.2c**, **7.EE.B.4a**). Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation (**7.RP.A.2d**).

In Topic C, students extend their reasoning about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers, such as a speed of $\frac{1}{2}$ mile per $\frac{1}{4}$ hour (**7.RP.A.1**). Students apply their experience in the first two topics and their new understanding of unit rates for ratios and rates involving fractions to solve multi-step ratio word problems (**7.RP.A.3**, **7.EE.B.4a**).

In the final topic of this module, students bring the sum of their experience with proportional relationships to the context of scale drawings (**7.RP.A.2b**, **7.G.A.1**). Given a scale drawing, students rely on their background in working with side lengths and areas of polygons (**6.G.A.1**, **6.G.A.3**) as they identify the scale factor as the constant of proportionality, calculate the actual lengths and areas of objects in the drawing, and create their own scale drawings of a two-dimensional view of a room or building. The topic culminates with a two-day experience of students creating a new scale drawing by changing the scale of an existing drawing.

Later in the year, in Module 4, students extend the concepts of this module to percent problems.

The module is composed of 22 lessons; 8 days are reserved for administering the Mid- and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Focus Standards

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks* $1/2$ *mile in each* $1/4$ *hour, compute the unit rate as the complex fraction* ${}^{1/2}/_{1/4}$ *miles per hour, equivalently* 2 *miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost* t *is proportional to the number* n *of items purchased at a constant price* p*, the relationship between the total cost and the number of items can be expressed as* $t = pn$*.*

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$, where r is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.4[2] Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is* 54 *cm. Its length is* 6 *cm. What is its width?*

[2]In this module, the equations are derived from ratio problems. 7.EE.B.4a is returned to in Modules 2 and 3.

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was* $2:1$*, because for every* 2 *wings there was* 1 *beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of* 3 *cups of flour to* 4 *cups of sugar, so there is* $3/4$ *cup of flour for each cup of sugar." "We paid* $\$75$ *for* 15 *hamburgers, which is a rate of* $\$5$ *per hamburger."*[3]

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole–number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took* 7 *hours to mow* 4 *lawns, then at that rate, how many lawns could be mowed in* 35 *hours? At what rate were lawns being mowed?*

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

[3]Expectations for unit rates in this grade are limited to non-complex fractions.

6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of and solve multi-step ratio problems, including cases with pairs of rational number entries; they use representations, such as ratio tables, the coordinate plane, and equations, and relate these representations to each other and to the context of the problem. Students depict the meaning of constant of proportionality in proportional relationships, the importance of $(0,0)$ and $(1, r)$ on graphs, and the implications of how scale factors magnify or shrink actual lengths of figures on a scale drawing.

MP.2 **Reason abstractly and quantitatively.** Students compute unit rates for paired data given in tables to determine if the data represents a proportional relationship. Use of concrete numbers will be analyzed to create and implement equations, including $y = kx$, where k is the constant of proportionality. Students decontextualize a given constant speed situation, representing symbolically the quantities involved with the formula, $\text{distance} = \text{rate} \times \text{time}$. In scale drawings, scale factors will be changed to create additional scale drawings of a given picture.

Terminology

New or Recently Introduced Terms

- **Constant of Proportionality** (If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the *constant of proportionality*. For example, if the ratio of y to x is 2 to 3, then the constant of proportionality is $\frac{2}{3}$, and $y = \frac{2}{3}x$.)
- **Miles per Hour** (One *mile per hour* is a proportional relationship between d miles and t hours given by the equation $d = 1 \cdot t$ (both d and t are positive real numbers). Similarly, for any positive real number v, v *miles per hour* is a proportional relationship between d miles and t hours given by $d = v \cdot t$. The unit for the rate, mile per hour (or mile/hour) is often abbreviated as mph.)
- **One-To-One Correspondence Between Two Figures in the Plane (description)** (For two figures in the plane, S and S', a *one-to-one correspondence between the figures* is a pairing between the points in S and the points in S' so that each point P of S is paired with one and only one point P' in S', and likewise, each point Q' in S' is paired with one and only one point Q in S.)

- **Proportional Relationship (description)** (A *proportional relationship* is a correspondence between two types of quantities such that the measures of quantities of the first type are proportional to the measures of quantities of the second type.

  Note that proportional relationships and ratio relationships describe the same set of ordered pairs but in two different ways. Ratio relationships are used in the context of working with equivalent ratios, while proportional relationships are used in the context of rates.)
- **Proportional To (description)** (Measures of one type of quantity are *proportional to* measures of a second type of quantity if there is a number k so that for every measure x of a quantity of the first type, the corresponding measure y of a quantity of the second type is given by kx; that is, $y = kx$. The number k is called the constant of proportionality.)
- **Scale Drawing and Scale Factor (description)** (For two figures in the plane, S and S', S' is said to be a *scale drawing* of S with *scale factor* r if there exists a one-to-one correspondence between S and S' so that, under the pairing of this one-to-one correspondence, the distance $|PQ|$ between any two points P and Q of S is related to the distance $|P'Q'|$ between corresponding points P' and Q' of S' by $|P'Q'| = r|PQ|$.)

Familiar Terms and Symbols[4]

- Equivalent Ratio
- Rate
- Ratio
- Ratio Table
- Unit Rate

Suggested Tools and Representations

- Ratio Table (See example below.)
- Coordinate Plane (See example below.)
- Equations of the Form $y = kx$

Ratio Table

Sugar	*Flour*
2	3
4	6
6	9

Coordinate Plane

[4]These are terms and symbols students have seen previously.

Preparing to Teach a Module

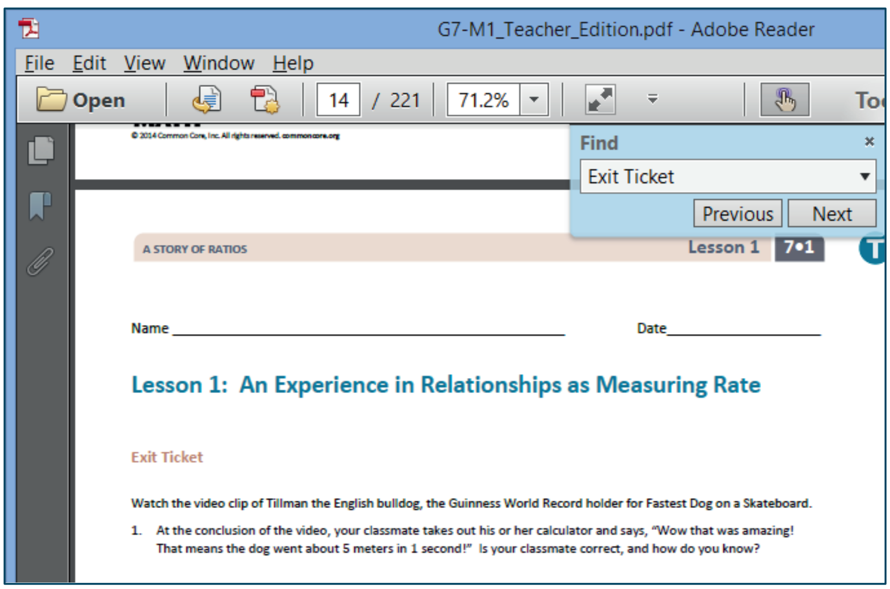
Preparation of lessons will be more effective and efficient if there has been an adequate analysis of the module first. Each module in *A Story of Ratios* can be compared to a chapter in a book. How is the module moving the plot, the mathematics, forward? What new learning is taking place? How are the topics and objectives building on one another? The following is a suggested process for preparing to teach a module.

Step 1: Get a preview of the plot.

A: Read the Table of Contents. At a high level, what is the plot of the module? How does the story develop across the topics?

B: Preview the module's Exit Tickets to see the trajectory of the module's mathematics and the nature of the work students are expected to be able to do.

Note: When studying a PDF file, enter "Exit Ticket" into the search feature to navigate from one Exit Ticket to the next.



Step 2: Dig into the details.

A: Dig into a careful reading of the Module Overview. While reading the narrative, liberally reference the lessons and Topic Overviews to clarify the meaning of the text – the lessons demonstrate the strategies, show how to use the models, clarify vocabulary, and build understanding of concepts.

B: Having thoroughly investigated the Module Overview, read through the Student Outcomes of each lesson (in order) to further discern the plot of the module. How do the topics flow and tell a coherent story? How do the outcomes move students to new understandings?

Step 3: Summarize the story.

Complete the Mid- and End-of-Module Assessments. Use the strategies and models presented in the module to explain the thinking involved. Again, liberally reference the lessons to anticipate how students who are learning with the curriculum might respond.

Rigor

Directions: While reading, annotate and consider your response to the following Guiding Question:

1. How might we incorporate the three components of rigor into our instruction?

Rigor: Pursue conceptual understanding, procedural skills and fluency, and application with equal intensity.

Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades. To help students meet the standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skills and fluency, and application.

> *Conceptual understanding*: The standards call for conceptual understanding of key concepts, such as place value and ratios. Students must be able to access concepts from a number of perspectives in order to see math as more than a set of mnemonics or discrete procedures.
>
> *Procedural skills and fluency*: The standards call for speed and accuracy in calculation. Students must practice core functions, such as single-digit multiplication, in order to have access to more complex concepts and procedures. Fluency must be addressed in the classroom or through supporting materials, as some students might require more practice than others.
>
> *Application*: The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency (NGA Center and CCSSO, "Key Shifts in Mathematics," 2010).

Lesson Types

Directions: While reading, annotate and consider your response to the following Guiding Question:

1. How might each Lesson Type relate to the components of rigor?

Problem Set Lesson: The teacher and students work through examples and complete exercises to develop or reinforce a concept.

Socratic Lesson: The teacher leads students in a conversation to develop a specific concept or proof.

Exploration Lesson: Students work independently or in small groups on a challenging problem followed by a debrief to clarify, expand, or develop math knowledge.

Modeling Cycle Lesson: Students practice all or part of the modeling cycle with real-world or mathematical problems that are either well- or ill-defined.

Directions: While reading, annotate and consider your response to the following Guiding Question:

1. What about *The Story of Eureka Math* most resonates with you? Why?

The Story of *Eureka Math*

What is *Eureka Math*?

Eureka Math is a coherent curriculum for Prekindergarten through Grade 12. Known as EngageNY early on,[2] the curriculum was written by teachers for teachers and partitioned into *A Story of Units®* for Prekindergarten through Grade 5, *A Story of Ratios®* for Grades 6 through 8, and *A Story of Functions®* for Grades 9 through 12.

How did *Eureka Math* come to be?

Lynne Munson, the CEO of Great Minds®, envisioned a rigorous PK–12 mathematics curriculum aligned with and built from the *Common Core State Standards for Mathematics* (NGA Center and CCSSO 2010) and the *Progressions for the Common Core State Standards in Mathematics* (Common Core Standards Writing Team 2011–2015) that accompany them. Scott Baldridge, a research mathematician with a deep understanding of and experience with elementary, middle, and high school mathematics, was uniquely qualified to mastermind that effort.[3]

Baldridge assembled a writing team of classroom teachers and math coaches from urban, low-income public and/or charter schools, rural schools, exclusive private schools, and universities. He also included mathematicians who would act as consultants and editors. What unified and continues to unify the diverse team are the following convictions:

- Math makes sense and is beautiful.
- Teaching math takes teamwork across the grade levels. Students can and do excel when their math experience is focused, coherent, and balanced in pursuit of procedural fluency, conceptual understanding, and application of understanding to solve real-world problems.
- Teaching math requires ongoing responsiveness to student work. Therefore, the expectation is that teachers will thoughtfully customize the *Eureka Math* curriculum to meet students' unique needs.

Let's take a moment to walk through an overview of each grade span, or story, of the *Eureka Math* curriculum for the purpose of exploring its coherence and, perhaps, getting a little excited about it.

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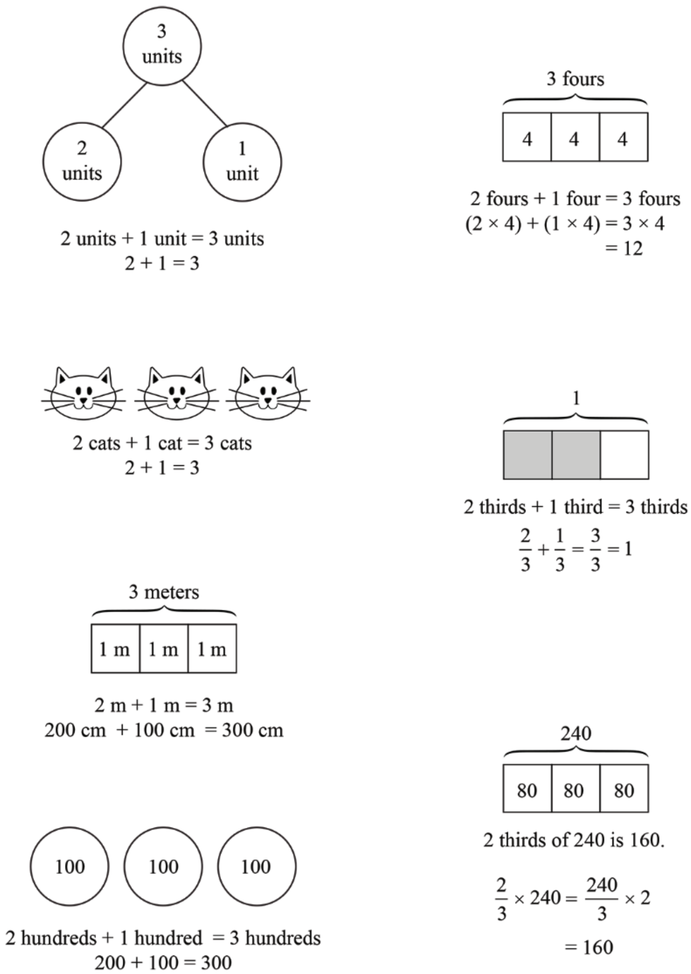
[2] Eureka Math *was created in partnership between the New York State Education Department (NYSED) and the nonprofit organization Great Minds. The state of New York used grant monies to fund the development of EngageNY, and when the contract with New York was fulfilled, Great Minds rebranded the curriculum as* Eureka Math.

[3] *Baldridge is coauthor with Thomas Parker of* Elementary Mathematics for Teachers (2004)*, rated by the National Commission on Teaching Quality (NCTQ) as the best preparation book for pre-service teachers.*

A Story of Units (Grades PK–5)

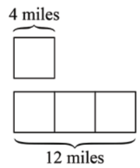


What units are in evidence in the examples above? What connections do you notice among the examples?

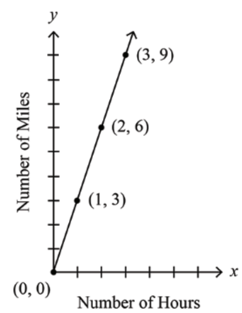
Notice how the clarity of the relationship between 2 cats + 1 cat, 2 meters + 1 meter, 2 hundreds + 1 hundred, 2 thirds + 1 third pops out at us. In *A Story of Units*, students begin in Kindergarten by adding and subtracting real-world units (e.g. apples, cats) and whole numbers up to 10. Their use of units culminates in Grade 5 with students using all four operations with multi-digit whole number, measurement, and fractional units; one arithmetic is used for all units. The focus on units makes coherence and connectedness readily apparent. Students are empowered to solve multi-step problems because they are constantly making sense of the math. For example, they make sense of how and why to break apart or put together units when using the four operations or how and why to apply the operations in word problems as they uncover and reason about part–whole relationships.

A Story of Ratios (Grades 6–8)

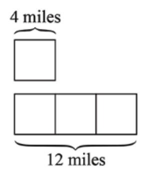
What connections do you notice between these examples and the math of Prekindergarten through Grade 5?



12 miles is 3 times as long as 4 miles.

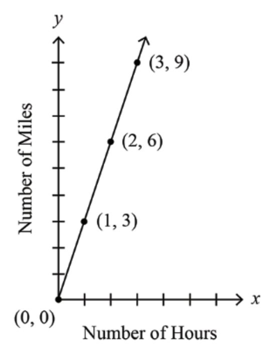


The unit rate is the value of the ratio of the number of miles to the number of hours. The unit rate in this example is 3.

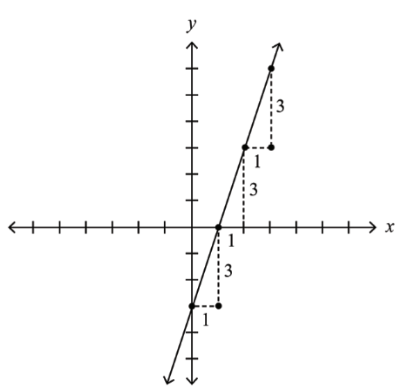


The ratio of 4 miles to 12 miles is 4 : 12 or 1 : 3.

Number of Hours, x	Number of Miles, y
0	0
1	3
2	6
3	9
x	$3x$



An infinite collection of equivalent ratios generates a line through the origin. The constant of proportionality is 3.



The slope of a line is the value of the ratio of the change in y to the change in x between any two points on the line. The slope of this line is 3.

Number of Yards	Number of Feet
0	0
1	3
2	6
3	9
4	12
y	$3y$

From Kindergarten through Grade 5, students work with different exchanges or conversions (e.g., 1 meter = 100 centimeters, 1 hundred = 100 ones, 1 whole = 3 thirds). When adding and subtracting base ten numbers, students exchange 10 tens to make 1 hundred or 10 tenths to make 1 whole—in essence, a ratio of 10 to 1.

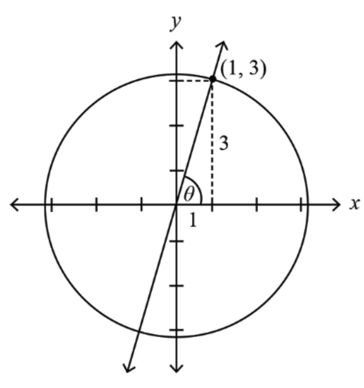
Also, students must understand the meaning of the factors in a product. For example, 3 fours can be written as 3×4. The factors can be interpreted as the number of units and the size of the unit, respectively. Students also work with measurement conversion, such as yards to feet as pictured at right.[4] Using the table, students see that the number of feet and the number of yards are related by a factor of 3; the ratio of feet to yards is 3 to 1. Understanding the meaning of the factor 3 is foundational to an understanding of unit rate in middle school. Consider the following example:

> Joey hikes at a constant speed of 6 miles every 2 hours. He hikes for 4 hours. How far does he hike?

The unit rate, 3, is the number of miles Joey hikes in 1 hour. A new measurement unit, miles per hour, is derived from the quotient of the number of miles to the duration of time: $\frac{6 \text{ miles}}{2 \text{ hours}} = \frac{6}{2}$ mph $=$ 3 mph. Unit rate sets the foundation for slope of a line as the ratio of the change in y to the change in x: $\frac{y_2 - y_1}{x_2 - x_1}$.

An understanding of the meaning of place value exchanges and measurement conversion leads to an understanding of ratio, unit rate, the constant of proportionality, scale factor, and slope. Understanding all of these multiplicative relationships sets the stage for understanding functions.

A Story of Functions (Grades 9–12)



The tangent of θ is the value of the ratio of the change in y to the change in x.

The tangent of θ shown above is 3.

What connections do you notice between the work of Prekindergarten through Grade 8 and the example at right?

The high school *Eureka Math* curriculum continues the story by building from the foundation set in *A Story of Units* and *A Story of Ratios*.

We have seen that an understanding of and skill with units leads to an understanding of and skill with ratios. An understanding of ratios leads to an understanding of slope; slope is the key to understanding linear functions and leads us to the derivative in calculus.

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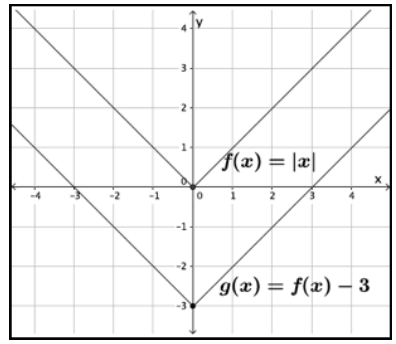
[4] *Students do not work explicitly with ratios in Prekindergarten through Grade 5, but the foundation is certainly being laid at a concrete, experiential level.*

In *A Story of Functions*, a solid grasp of linear functions supports understanding characteristics of other common functions, for example:



- In Algebra I, students study functions with the domain of all real numbers and expand their experience to include exponential, quadratic, square root, and cube root functions. They also study piecewise-defined functions, including absolute value and step functions.
- In Geometry, students continue their study of transformations of the plane, which began in Grade 8.
- In Algebra II, students study inverse functions with the primary intent of introducing logarithmic functions as inverses of exponential functions.
- In Precalculus and Advanced Topics, students continue to explore the properties of trigonometric functions that began in Algebra II and use the definitions of sine and cosine and the properties of the unit circle to prove the addition and subtraction formulas for sine, cosine, and tangent. Students also explore rational functions, inverse functions, and the inverse trigonometric functions.

Students move from working with quantities (i.e., a number with a unit such as 3 feet) to working with ratios (i.e., a relationship between two quantities such as 3 miles to 1 hour) to working with functions of rational and real numbers. A function relates every number in its domain to another number in its range in a prescribed way. When we study ratios, we generally see the relationship only between the values presented in a table; using a function allows us to extend this relationship to many more values—perhaps an infinite number of values.

Just as the events of a novel are in a specific order for good reason, the three parts of the *Eureka Math* curriculum (*A Story of Units, A Story of Ratios, A Story of Functions*) build on one another logically. After all, mathematics truly is a unified, deeply interconnected body of knowledge, and the teacher–writers of *Eureka Math* believe it is best presented and taught that way.

Directions: While reading, annotate and consider your response to the following Guiding Question:

1. Why might each step be important to your instructional preparation?

EUREKA MATH™

Preparing to Teach a Lesson
A Story of Ratios® (6–8)
A Story of Functions® (9–12)

We recommend a three-step process to prepare a lesson. The process considers that, at times, teachers may need to adjust (customize) lessons to fit the time constraints and unique needs of their students.

1. Discern the Plot

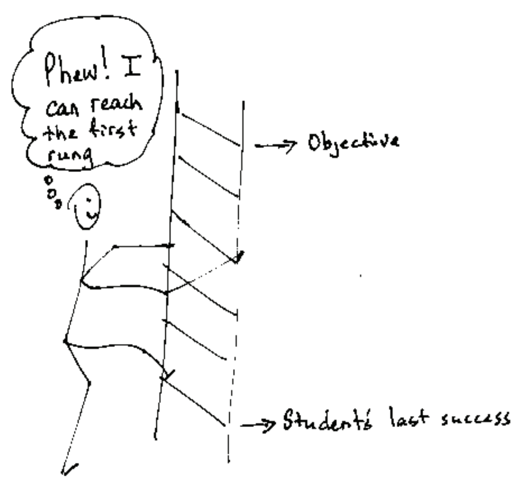
A. Briefly review the module's Contents, recalling the overall story of the module and analyzing the role of this lesson in the module.

B. Read the Topic Overview that relates to the lesson, and then review the Student Outcomes and Exit Ticket for each lesson in the topic.

C. Review the assessment following the topic, keeping in mind that assessments occur midway through and at the end of the module.

2. Find the Ladder

The ladder is a metaphor for the teaching sequence. At the macro level, the sequence is evident in the role this lesson plays in the overall story, and at the lesson level, each rung on the ladder represents the next step in understanding or the next skill students need to reach the objective.



A. Work through the lesson, answering and completing each question, example, exercise, and challenge.

B. Analyze the new complexities or concepts each question or problem introduces. These notes on the sequence of new complexities and concepts are the rungs of the ladder.

C. Anticipate where students might stumble, and note the potential cause of the struggle.

D. Answer the Closing questions, anticipating how students will respond.

3. Hone the Lesson

Lessons may require customization if the class period is not long enough to complete the lesson, and/or students lack prerequisite skills and understanding to move through the entire lesson in the time allotted. When customizing the lesson, first designate problems and questions as Must Do, Could Do, and/or Extension problems.

A. Select Must Do problems that meet the Student Outcomes and provide students with a coherent experience. (Think about the rungs on a ladder.) The expectation is that most of the class can complete the Must Do problems within the allotted time. When choosing the Must Do problems, keep in mind the need for a balance of dialogue and conceptual questioning, application problems, and abstract problems as well as a balance in students' use of pictorial or graphical representations and abstract representations. Highlight dialogue to include in instruction so students have a chance to articulate and consolidate understanding as they move through the lesson.

B. Must Do problems might also include remedial work as necessary for the whole class, a small group, or individual students. Depending on the anticipated difficulties, remedial work might take on different forms as the chart below suggests.

Anticipated Difficulty	Must Do Remedial Problem Suggestion
The first problem of the lesson is too challenging.	Write a short sequence of problems on the board that provides a ladder to Problem 1. Direct students to complete those first problems to empower them to begin the lesson.
The jump in complexity between two problems is too big.	Provide a problem or set of problems that bridges student understanding between the two problems.
Students lack fluency or foundational skills necessary for the lesson.	Before beginning the lesson, do a quick, engaging fluency exercise, such as a Rapid Whiteboard Exchange or Sprint. Before beginning any fluency activity, assess whether students have conceptual understanding of the problems in the set and whether they will have success with the easiest problem in the set.
Students need more work at the concrete or pictorial level.	Provide manipulatives or the opportunity for students to draw solution strategies.
Students need more work at the abstract level.	Add a Rapid Whiteboard Exchange of abstract problems for students to complete toward the end of the lesson.

C. Could Do problems are for students who work with greater fluency and understanding and can therefore complete more work within a given time frame.

D. Occasionally designate a particularly complex problem as an Extension problem for advanced students. Consider creating the opportunity for students to share extension solutions with the class.

E. For customized lessons, be sure to select Closing questions that reflect such decisions and adjust the Exit Ticket as necessary.

Lesson Demonstration

Directions

1. As the facilitator demonstrates G7 M3 L16, follow along by using the student pages in the Virtual Engagement Materials, Pages 27-33.
2. As the facilitator models the lesson, think about the following: How are the components of rigor embedded in the lesson?

<u>Conceptual Understanding</u>:

<u>Procedural Skills and Fluency:</u>

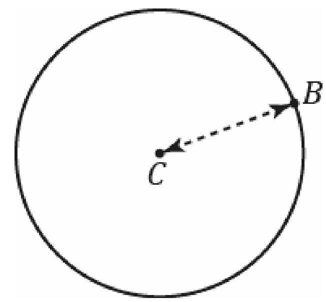
<u>Application:</u>

Lesson 16: The Most Famous Ratio of All

Classwork

Opening Exercise

a. Using a compass, draw a circle like the picture to the right.



C is the *center* of the circle.
The distance between C and B is the *radius* of the circle.

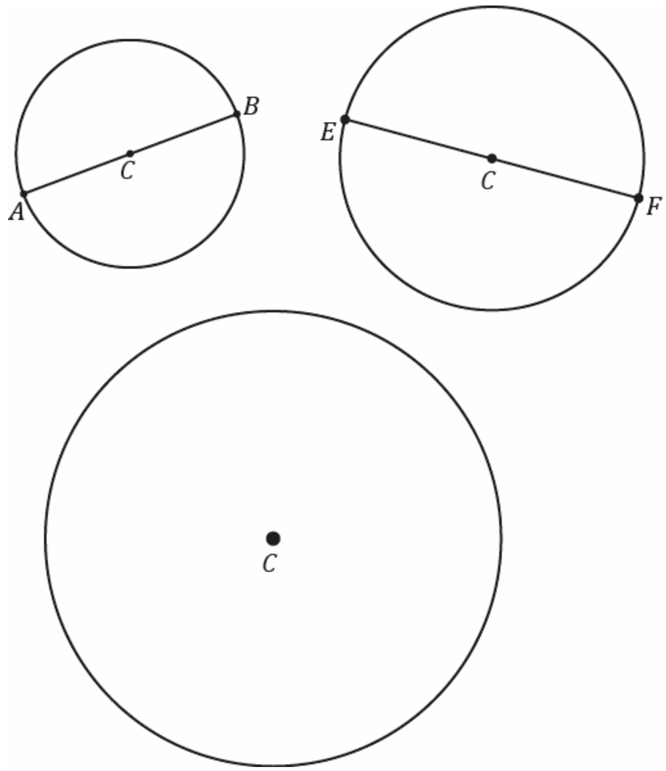
b. Write your own definition for the term *circle*.

c. Extend segment CB to a segment AB in part (a), where A is also a point on the circle.

The length of the segment AB is called the diameter of the circle.

d. The diameter is _______________________ as long as the radius.

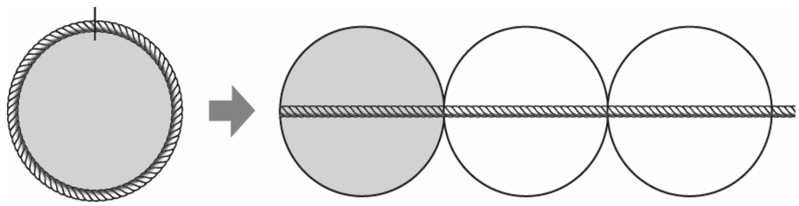
e. Measure the radius and diameter of each circle. The center of each circle is labeled C.



f. Draw a circle of radius 6 cm.

Mathematical Modeling Exercise

The ratio of the circumference to its diameter is always the same for any circle. The value of this ratio, $\frac{\text{Circumference}}{\text{Diameter}}$, is called the number *pi* and is represented by the symbol π.

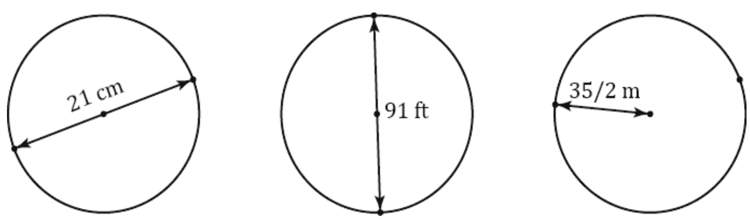


Since the circumference is a little greater than 3 times the diameter, π is a number that is a little greater than 3. Use the symbol π to represent this special number. Pi is a non-terminating, non-repeating decimal, and mathematicians use the symbol π or approximate representations as more convenient ways to represent pi.

- $\pi \approx 3.14$ or $\frac{22}{7}$.
- The ratios of the circumference to the diameter and $\pi : 1$ are equal.
- $\text{Circumference of a Circle} = \pi \times \text{Diameter}$.

Example

a. The following circles are not drawn to scale. Find the circumference of each circle. (Use $\frac{22}{7}$ as an approximation for π.)

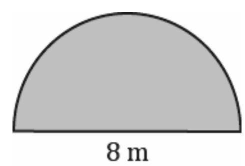


b. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest tenth. (Use 3.14 as an approximation for π.)

c. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest hundredth. (Use the π button on your calculator as an approximation for π.)

d. A circle has a radius of r cm and a circumference of C cm. Write a formula that expresses the value of C in terms of r and π.

e. The figure below is in the shape of a semicircle. A semicircle is an arc that is half of a circle. Find the perimeter of the shape. (Use 3.14 for π.)



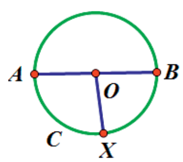
Relevant Vocabulary

CIRCLE: Given a point O in the plane and a number $r > 0$, the *circle with center* O *and radius* r is the set of all points in the plane whose distance from the point O is equal to r.

RADIUS OF A CIRCLE: The *radius* is the length of any segment whose endpoints are the center of a circle and a point that lies on the circle.

DIAMETER OF A CIRCLE: The *diameter of a circle* is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If r is the *radius* of a circle, then the diameter is $2r$.

The word *diameter* can also mean the segment itself. Context determines how the term is being used: *The diameter* usually refers to the length of the segment, while *a diameter* usually refers to a segment. Similarly, *a radius* can refer to a segment from the center of a circle to a point on the circle.

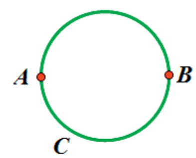


Circle C

Radii: $\overline{OA}$, $\overline{OB}$, $\overline{OX}$

Diameter: $\overline{AB}$

Circumference

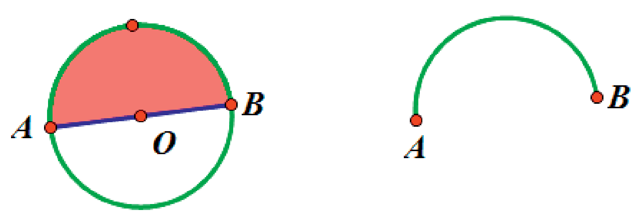


CIRCUMFERENCE: The circumference of a circle is the distance around a circle.

PI: The number *pi*, denoted by π, is the value of the ratio given by the circumference to the diameter, that is $\pi = \frac{\text{circumference}}{\text{diameter}}$. The most commonly used approximations for π is 3.14 or $\frac{22}{7}$.

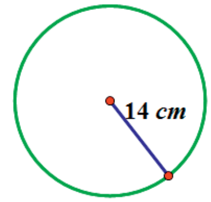
SEMICIRCLE: Let C be a circle with center O, and let A and B be the endpoints of a diameter. A *semicircle* is the set containing A, B, and all points that lie in a given half-plane determined by $\overline{AB}$ (diameter) that lie on circle C.

Semicircle



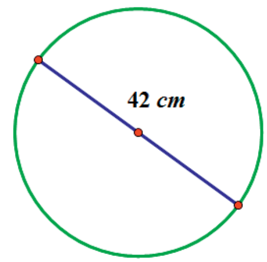
Problem Set

1. Find the circumference.



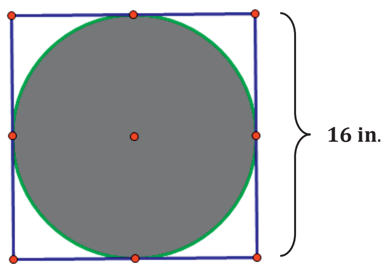
   a. Give an exact answer in terms of π.
   b. Use $\pi \approx \frac{22}{7}$ and express your answer as a fraction in lowest terms.
   c. Use the π button on your calculator, and express your answer to the nearest hundredth.

   Find the circumference.

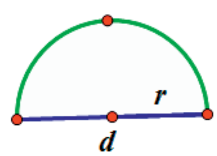


   d. Give an exact answer in terms of π.
   e. Use $\pi \approx \frac{22}{7}$, and express your answer as a fraction in lowest terms.

2. The figure shows a circle within a square. Find the circumference of the circle. Let $\pi \approx 3.14$.

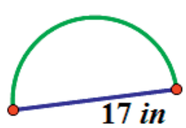


3. Consider the diagram of a semicircle shown.



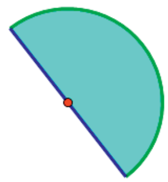
   a. Explain in words how to determine the perimeter of a semicircle.
   b. Using d to represent the diameter of the circle, write an algebraic equation that will result in the perimeter of a semicircle.
   c. Write another algebraic equation to represent the perimeter of a semicircle using r to represent the radius of a semicircle.

4. Find the perimeter of the semicircle. Let $\pi \approx 3.14$.

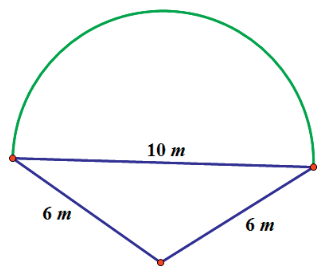


5. Ken's landscape gardening business makes odd-shaped lawns that include semicircles. Find the length of the edging material needed to border the two lawn designs. Use 3.14 for π.

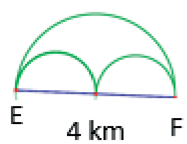
   a. The radius of this flower bed is 2.5 m.



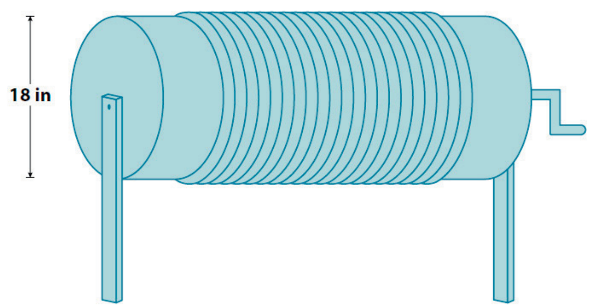
   b. The diameter of the semicircular section is 10 m, and the lengths of the sides of the two sides are 6 m.



6. Mary and Margaret are looking at a map of a running path in a local park. Which is the shorter path from E to F, along the two semicircles or along the larger semicircle? If one path is shorter, how much shorter is it? Let $\pi \approx 3.14$.



7. Alex the electrician needs 34 yards of electrical wire to complete a job. He has a coil of wiring in his workshop. The coiled wire is 18 inches in diameter and is made up of 21 circles of wire. Will this coil be enough to complete the job? Let $\pi \approx 3.14$.



Search and Find Note Catcher

Directions

1. Collaboratively look through the Teacher Edition from our Lesson Demonstration, Pages 36-46.
2. Record the page number of each item you find for reference when we debrief.
3. Save the last column for taking notes as we debrief each component as a group.

Look for	Page Number	Notes
Student Outcomes		
Lesson with a Lesson Note		
Example Followed by an Exercise		
Closed Bullet Followed by an Open Bullet		
Reference to a Mathematical Practice Standard		

Look for	Page Number	Notes
Box around an Item		
Scaffolding Box		
Closing		
Lesson Summary		
Blank Exit Ticket		
Exit Ticket Sample Solution		
Problem Set Sample Solutions		

Lesson 16: The Most Famous Ratio of All

Student Outcomes

- Students develop the definition of a circle using diameter and radius.
- Students know that the distance around a circle is called the *circumference* and discover that the ratio of the circumference to the diameter of a circle is a special number called pi, written π.
- Students know the formula for the circumference C of a circle, of diameter d, and radius r. They use scale models to derive these formulas.
- Students use $\frac{22}{7}$ and 3.14 as estimates for π and informally show that π is slightly greater than 3.

Lesson Notes

Although students were introduced to circles in kindergarten and worked with angles and arcs measures in Grades 4 and 5, they have not examined a precise definition of a circle. This lesson combines the definition of a circle with the application of constructions with a compass and straightedge to examine the ideas associated with circles and circular regions.

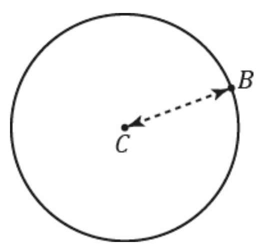
Classwork

Opening Exercise (10 minutes)

Materials: Each student has a compass and metric ruler.

Opening Exercise

a. **Using a compass, draw a circle like the picture to the right.**



C is the *center* of the circle.
The distance between C and B is the *radius* of the circle.

b. **Write your own definition for the term *circle*.**

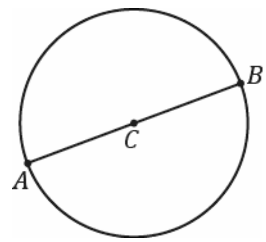
Student responses will vary. Many might say, "It is round." "It is curved." "It has an infinite number of sides." "The points are always the same distance from the center." Analyze their definitions, showing how other figures such as ovals are also "round" or "curved." Ask them what is special about the compass they used. (Answer: The distance between the spike and the pencil is fixed when drawing the circle.) Let them try defining a circle again with this new knowledge.

Present the following information about a circle.

- **CIRCLE:** Given a point O in the plane and a number $r > 0$, the *circle with center O and radius r* is the set of all points in the plane whose distance from the point O is equal to r.

- What does the distance between the spike and the pencil on a compass represent in the definition above?
  - *The radius* r
- What does the spike of the compass represent in the definition above?
  - *The center* C
- What does the image drawn by the pencil represent in the definition above?
  - *The set of all points*

c. Extend segment CB to a segment AB, where A is also a point on the circle.



The length of the segment AB is called the diameter of the circle.

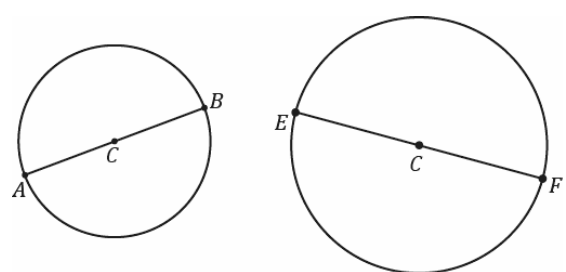
d. The diameter is *twice, or* 2 *times,* as long as the radius.

MP.3 After each student measures and finds that the diameter is twice as long as the radius, display several student examples of different-sized circles to the class. Did everyone get a measure that was twice as long? Ask if a student can use the definition of a circle to explain why the diameter must be twice as long.

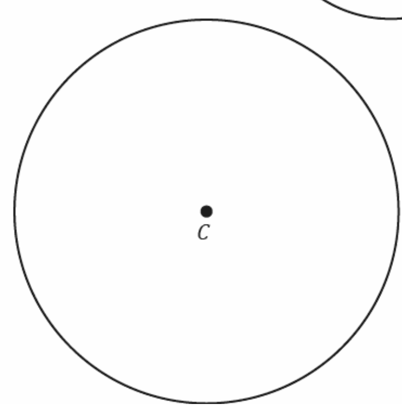
e. Measure the radius and diameter of each circle. The center of each circle is labeled C.

$CB = 1.5$ cm, $AB = 3$ cm, $CF = 2$ cm, $EF = 4$ cm

The radius of the largest circle is 3 cm. *The diameter is* 6 cm.



f. Draw a circle of radius 6 cm.



Part (f) may not be as easy as it seems. Let students grapple with how to measure 6 cm with a compass. One difficulty they might encounter is trying to measure 6 cm by putting the spike of the compass on the edge of the ruler (i.e., the 0 cm mark). Suggest either of the following: (1) Measure the compass from the 1 cm mark to the 7 cm mark, or (2) Mark two points 6 cm apart on the paper first; then, use one point as the center.

Mathematical Modeling Exercise (15 minutes)

Materials: a bicycle wheel (as large as possible), tape or chalk, a length of string long enough to measure the circumference of the bike wheel

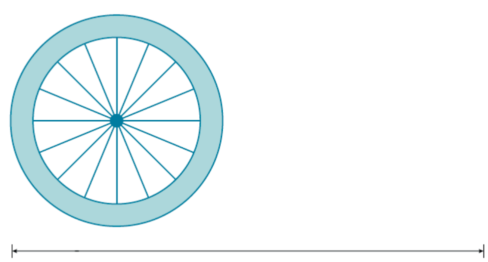
Activity: Invite the entire class to come up to the front of the room to measure a length of string that is the same length as the distance around the bicycle wheel. Give them the tape or chalk and string, but *do not tell them how to use these materials to measure the circumference*, at least not yet. The goal is to set up several "ah-ha" moments for students. Give them time to *try* to wrap the string around the bicycle wheel. They will quickly find that this way of trying to measure the circumference is unproductive (the string will pop off). Lead them to the following steps for measuring the circumference, even if they do succeed with wrapping the string:

1. Mark a point on the wheel with a piece of masking tape or chalk.
2. Mark a starting point on the floor, align it with the mark on the wheel, and carefully roll the wheel so that it rolls one complete revolution.
3. Mark the endpoint on the floor with a piece of masking tape or chalk.

Dramatically walk from the beginning mark to the ending mark on the floor, declaring, "The length between these two marks is called the *circumference* of the wheel; it is the distance around the wheel. We can now easily measure that distance with string." First, ask two students to measure a length of string using the marks; then, ask them to hold up the string directly above the marks in front of the rest of the class. Students are ready for the next "ah-ha" moment.

- Why is this new way of measuring the string better than trying to wrap the string around the wheel? (Because it leads to an accurate measurement of the circumference.)
- The circumference of any circle is always the same multiple of the diameter. Mathematicians call this number *pi*. It is one of the few numbers that is so special it has its own name. Let's see if we can estimate the value of pi.

Take the wheel and carefully measure three diameter lengths using the wheel itself, as in the picture below.



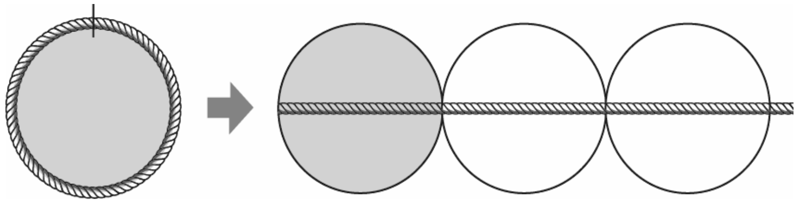
Mark the three diameter lengths on the rope with a marker. Then, have students wrap the rope around the wheel itself.

If the circumference was measured carefully, students see that the string is three wheel diameters plus *a little bit extra* at the end. Have students estimate how much the extra bit is; guide them to report, "It's a little more than a tenth of the bicycle diameter."

- The circumference of any circle is a little more than 3 times its diameter. The number pi is a little greater than 3.
- Use the symbol π to represent this special number. Pi is a non-terminating, non-repeating decimal, and mathematicians use the symbol π or approximate representations as more convenient ways to represent pi.

Mathematical Modeling Exercise

The ratio of the circumference to its diameter is always the same for any circle. The value of this ratio, $\frac{\text{Circumference}}{\text{Diameter}}$, is called the number *pi* and is represented by the symbol π.



Since the circumference is a little greater than 3 times the diameter, π is a number that is a little greater than 3. Use the symbol π to represent this special number. Pi is a non-terminating, non-repeating decimal, and mathematicians use the symbol π or approximate representations as more convenient ways to represent pi.

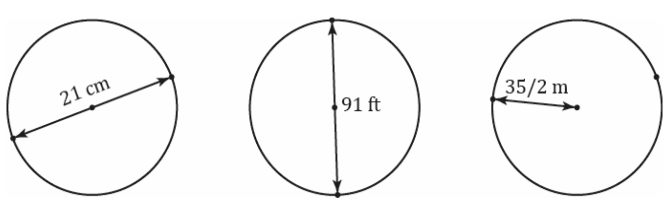
- $\pi \approx 3.14$ **or** $\frac{22}{7}$.
- **The ratios of the circumference to the diameter and $\pi : 1$ are equal.**
- $\textbf{Circumference of a Circle} = \pi \times \textbf{Diameter}$.

Example (10 minutes)

Note that both 3.14 and $\frac{22}{7}$ are excellent approximations to use in the classroom: One helps students' fluency with decimal number arithmetic, and the second helps students' fluency with fraction arithmetic. After learning about π and its approximations, have students use the π button on their calculators as another approximation for π. Students should use all digits of π in the calculator and round appropriately.

Example

a. **The following circles are not drawn to scale. Find the circumference of each circle. (Use $\frac{22}{7}$ as an approximation for π.)**



66 cm; 286 ft.; 110 m; *Ask students if these numbers are roughly three times the diameters.*

b. **The radius of a paper plate is 11.7 cm. Find the circumference to the nearest tenth. (Use 3.14 as an approximation for π.)**

Diameter: 23.4 cm*; circumference:* 73.5 cm

Extension for this problem: Bring in paper plates, and ask students how to find the center of a paper plate. This is not as easy as it sounds because the center is not given. Answer: Fold the paper plate in half twice. The intersection of the two folds is the center. Afterward, have students fold their paper plates several more times. Explore what happens. Ask students why the intersection of both lines is guaranteed to be the center. Answer: The first fold guarantees that the crease is a diameter, the second fold divides that diameter in half, but the midpoint of a diameter is the center.

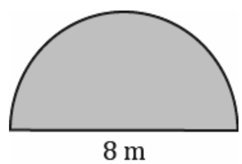
c. **The radius of a paper plate is 11.7 cm. Find the circumference to the nearest hundredth. (Use the π button on your calculator as an approximation for π.)**

Circumference: 73.51 cm

d. **A circle has a radius of r cm and a circumference of C cm. Write a formula that expresses the value of C in terms of r and π.**

$C = \pi \cdot 2r$, *or* $C = 2\pi r$.

e. **The figure below is in the shape of a semicircle. A semicircle is an arc that is half of a circle. Find the perimeter of the shape. (Use 3.14 for π.)**



$8\text{ m} + \frac{8(3.14)}{2}\text{ m} = 20.56\text{ m}$

Closing (5 minutes)

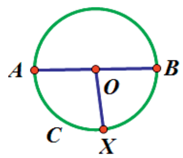
Relevant Vocabulary

CIRCLE: Given a point O in the plane and a number $r > 0$, the *circle with center O and radius r* is the set of all points in the plane whose distance from the point O is equal to r.

RADIUS OF A CIRCLE: The radius is the length of any segment whose endpoints are the center of a circle and a point that lies on the circle.

DIAMETER OF A CIRCLE: The *diameter of a circle* is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If r is the *radius* of a circle, then the diameter is $2r$.

The word *diameter* can also mean the segment itself. Context determines how the term is being used: *The diameter* usually refers to the length of the segment, while *a diameter* usually refers to a segment. Similarly, *a radius* can refer to a segment from the center of a circle to a point on the circle.

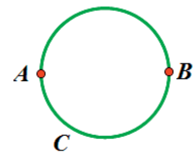


Circle C

Radii: $\overline{OA}, \overline{OB}, \overline{OX}$

Diameter: $\overline{AB}$

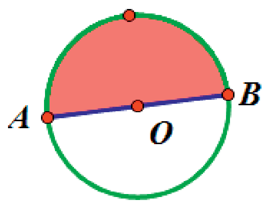
Circumference



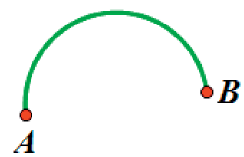
CIRCUMFERENCE: The circumference of a circle is the distance around a circle.

PI: The number *pi*, denoted by π, is the value of the ratio given by the circumference to the diameter, that is $\pi = \frac{\text{circumference}}{\text{diameter}}$. The most commonly used approximations for π is 3.14 or $\frac{22}{7}$.

SEMICIRCLE: Let C be a circle with center O, and let A and B be the endpoints of a diameter. A *semicircle* is the set containing A, B, and all points that lie in a given half-plane determined by $\overline{AB}$ (diameter) that lie on circle C.



Semicircle



Exit Ticket (5 minutes)

The Exit Ticket calls on students to synthesize their knowledge of circles and rectangles. A simpler alternative is to have students sketch a circle with a given radius and then have them determine the diameter and circumference of that circle.

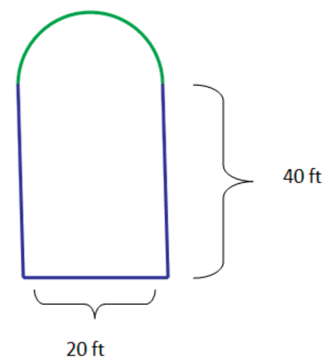
Name ____ Date____________________

Lesson 16: The Most Famous Ratio of All

Exit Ticket

Brianna's parents built a swimming pool in the backyard. Brianna says that the distance around the pool is 120 feet.

1. Is she correct? Explain why or why not.

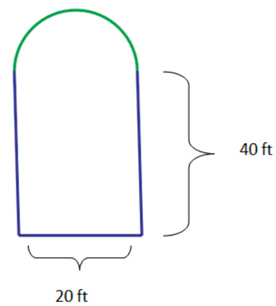


2. Explain how Brianna would determine the distance around the pool so that her parents would know how many feet of stone to buy for the edging around the pool.

3. Explain the relationship between the circumference of the semicircular part of the pool and the width of the pool.

Exit Ticket Sample Solutions

Brianna's parents built a swimming pool in the backyard. Brianna says that the distance around the pool is 120 feet.



1. Is she correct? Explain why or why not.

   Brianna is incorrect. The distance around the pool is 131.4 ft. She found the distance around the rectangle only and did not include the distance around the semicircular part of the pool.

2. Explain how Brianna would determine the distance around the pool so that her parents would know how many feet of stone to buy for the edging around the pool.

   In order to find the distance around the pool, Brianna must first find the circumference of the semicircle, which is $C = \frac{1}{2} \cdot \pi \cdot 20$ ft., or 10π ft., or about 31.4 ft. The sum of the three other sides is $20 \text{ ft.} + 40 \text{ ft.} + 40 \text{ ft.} = 100 \text{ ft.}$; the perimeter is $100 \text{ ft.} + 31.4 \text{ ft.} = 131.4 \text{ ft.}$

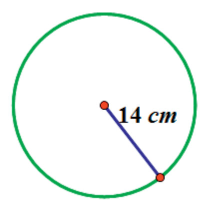
3. Explain the relationship between the circumference of the semicircular part of the pool and the width of the pool.

   The relationship between the circumference of the semicircular part and the width of the pool is the same as half of π because this is half the circumference of the entire circle.

Problem Set Sample Solutions

Students should work in cooperative groups to complete the tasks for this exercise.

1. Find the circumference.



   a. Give an exact answer in terms of π.

   $C = 2\pi r$
   $C = 2\pi \cdot 14 \text{ cm}$
   $C = 28\pi \text{ cm}$

   b. Use $\pi \approx \frac{22}{7}$, and express your answer as a fraction in lowest terms.

   $C \approx 2 \cdot \frac{22}{7} \cdot 14 \text{ cm}$
   $C \approx 88 \text{ cm}$

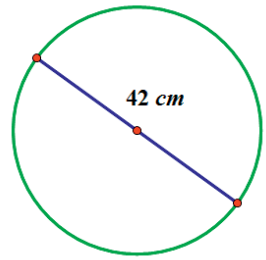
   c. Use the π button on your calculator, and express your answer to the nearest hundredth.

   $C = 2 \cdot \pi \cdot 14 \text{ cm}$

   $C \approx 87.96 \text{ cm}$

2. Find the circumference.

   a. Give an exact answer in terms of π.

   $d = 42$ cm

   $C = \pi d$

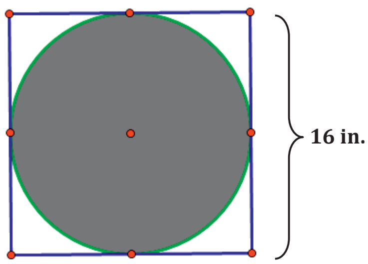
   $C = 42\pi$ cm

   b. Use $\pi \approx \frac{22}{7}$, and express your answer as a fraction in lowest terms.

   $C \approx 42 \text{ cm} \cdot \frac{22}{7}$

   $C \approx 132$ cm

3. The figure shows a circle within a square. Find the circumference of the circle. Let $\pi \approx 3.14$.



The diameter of the circle is the same as the length of the side of the square.

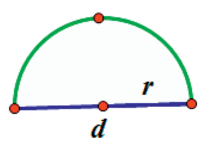
$C = \pi d$

$C = \pi \cdot 16$ in.

$C \approx 3.14 \cdot 16$ in.

$C \approx 50.24$ in.

4. Consider the diagram of a semicircle shown.

   a. Explain in words how to determine the perimeter of a semicircle.

   The perimeter is the sum of the length of the diameter and half of the circumference of a circle with the same diameter.

   b. Using d to represent the diameter of the circle, write an algebraic equation that will result in the perimeter of a semicircle.

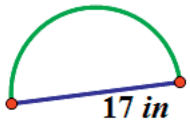
   $P = d + \frac{1}{2}\pi d$

   c. Write another algebraic equation to represent the perimeter of a semicircle using r to represent the radius of a semicircle.

   $P = 2r + \frac{1}{2}\pi \cdot 2r$

   $P = 2r + \pi r$

5. Find the perimeter of the semicircle. Let $\pi \approx 3.14$.



$P = d + \frac{1}{2}\pi d$

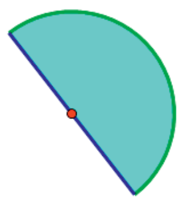
$P \approx 17 \text{ in.} + \frac{1}{2} \cdot 3.14 \cdot 17 \text{ in.}$

$P \approx 17 \text{ in.} + 26.69 \text{ in.}$

$P \approx 43.69 \text{ in.}$

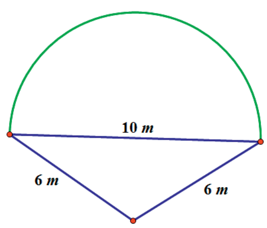
6. Ken's landscape gardening business makes odd-shaped lawns that include semicircles. Find the length of the edging material needed to border the two lawn designs. Use 3.14 for π.

a. The radius of this flowerbed is 2.5 m.



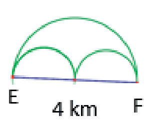
A semicircle has half of the circumference of a circle. If the circumference of the semicircle is $C = \frac{1}{2}(\pi \cdot 2 \cdot 2.5 \text{ m})$, then the circumference approximates 7.85 m. The length of the edging material must include the circumference and the diameter; $7.85 \text{ m} + 5 \text{ m} = 12.85 \text{ m}$. Ken needs 12.85 meters of edging to complete his design.

b. The diameter of the semicircular section is 10 m, and the lengths of the two sides are 6 m.



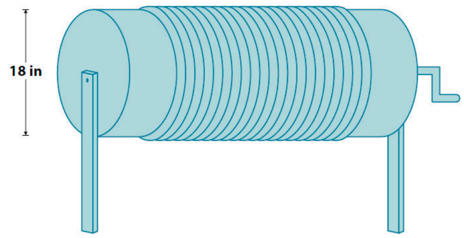
The circumference of the semicircular part has half of the circumference of a circle. The circumference of the semicircle is $C = \frac{1}{2}\pi \cdot 10 \text{ m}$, which is approximately 15.7 m. The length of the edging material must include the circumference of the semicircle and the perimeter of two sides of the triangle;
$15.7 \text{ m} + 6 \text{ m} + 6 \text{ m} = 27.7 \text{ m}$. *Ken needs 27.7 meters of edging to complete his design.*

7. Mary and Margaret are looking at a map of a running path in a local park. Which is the shorter path from E to F, along the two semicircles or along the larger semicircle? If one path is shorter, how much shorter is it? Let $\pi \approx 3.14$.



A semicircle has half of the circumference of a circle. The circumference of the large semicircle is $C = \frac{1}{2}\pi \cdot 4 \text{ km}$, or 6.28 km. The diameter of the two smaller semicircles is 2 km. The total circumference would be the same as the circumference for a whole circle with the same diameter. If $C = \pi \cdot 2 \text{ km}$, then $C = 6.28$ km. The distance around the larger semicircle is the same as the distance around both of the semicircles. So, both paths are equal in distance.

8. **Alex the electrician needs 34 yards of electrical wire to complete a job. He has a coil of wiring in his workshop. The coiled wire is 18 inches in diameter and is made up of 21 circles of wire. Will this coil be enough to complete the job? Let $\pi \approx 3.14$.**



The circumference of the coil of wire is $C = \pi \cdot 18 \text{ in.}$, or approximately 56.52 in. If there are 21 circles of wire, then the number of circles times the circumference will yield the total number of inches of wire in the coil. If $56.52 \text{ in.} \cdot 21 \approx 1186.92 \text{ in.}$, then $\frac{1186.92 \text{ in.}}{36 \text{ in.}} \approx 32.97 \text{ yd.}$ ($1 \text{ yd.} = 3 \text{ ft.} = 36 \text{ in.}$ When converting inches to yards, you must divide the total inches by the number of inches in a yard, which is 36 inches.) Alex will not have enough wire for his job in this coil of wire.

Lesson Preparation Template

Lesson Number: **Title:** **Day/Date:**	
Lesson Type (circle one): Problem Set · Exploration · Socratic · Modeling Cycle	
Student Outcomes	
Materials	
Fluency (as needed)	
Classwork Key ▪ Must Do (M) ▪ Could Do (C) ▪ Extension Problems (E)	

Closing	
Formative Assessment (e.g., Exit Ticket)	
Problem Set	
Lesson Notes	

Closing Debrief

Directions

1. To reflect on the day, return to the start of your Virtual Engagement Materials
2. Silently browse the Virtual Engagement Materials, reconstructing what we did during our time together. Look for the flow of the session's logic. (3–5 minutes)
3. Jot down takeaways below.

Resources

Eureka Math Website

www.greatminds.org

Eureka Math Blogs

eurmath.link/blog

Eureka Math Facebook

https://www.facebook.com/eurekamathofficial

Eureka Math Pinterest

https://www.pinterest.com/eurekamath0130/

Eureka Math Twitter

https://twitter.com/eureka_math

Eureka Math Champions

https://greatminds.org/math/champions

Progressions Documents

http://ime.math.arizona.edu/progressions/

Works Cited

Common Core Standards Writing Team. 2011–2015. *Progressions for the Common Core State Standards in Mathematics*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. http://math.arizona.edu/~ime/progressions/.

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Parker, Thomas H., and Scott J. Baldridge. 2004. *Elementary Mathematics for Teachers*. Okemos, MI: Sefton-Ash.