



***Launch Eureka Math:
A Story of Ratios®
A Story of Functions®***

Sample Work

Sample Work

Coherence from Kindergarten through Precalculus and Advanced Topics

Directions

Analyze the following expressions, and then share with a partner: What might be the value of writing or saying the unit as part of the expression?

5 cars + 3 cars

5 tens + 3 tens

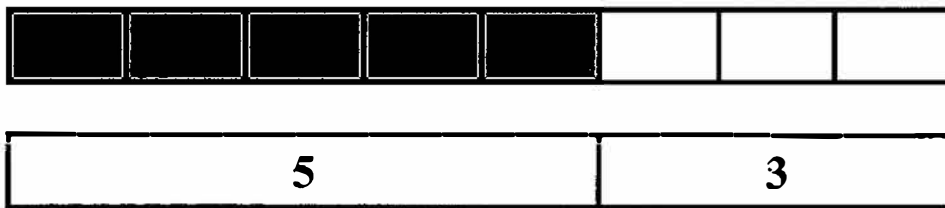
5 meters + 3 meters

5 fours + 3 fours

5 ninths + 3 ninths

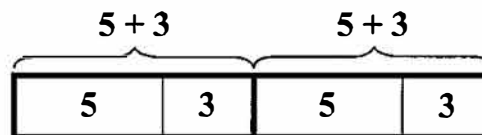
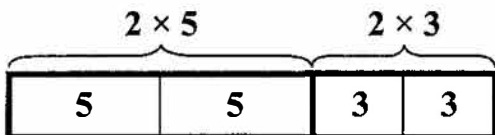
$5x + 3x = \underline{8x}$ ← UNIT OF UNKNOWN VALUE

NEVER THOUGHT OF DENOMINATORS AS UNITS



EACH UNIT OF ONE IS NOT PARTITIONED

Use the tape diagrams to answer the following questions.



How many fives are in the model? 2
 How many threes are in the model? 2

How many fives are in the model? 2
 How many threes are in the model? 2

Write at least one expression to represent each model.

$2(5) + 2(3)$

$2(5+3)$

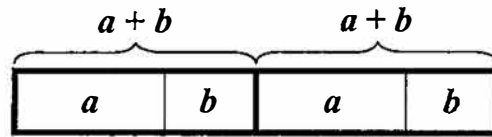
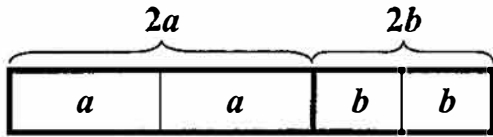
Write down ways of representing each expression with words.

TWO TIMES THE QUANTITY OF FIVE PLUS TWO TIMES THE QUANTITY OF THREE

TWO TIMES THE QUANTITY OF FIVE PLUS THREE

Grade 6

a. Use the model to answer the following questions.



How many a 's are in the model? 2
 How many b 's are in the model? 2

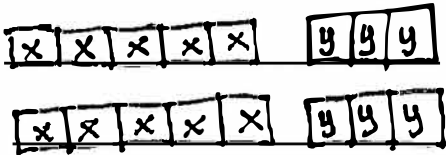
How many a 's are in the model? 2
 How many b 's are in the model? 2

Write at least one expression to represent each model.

$2a + 2b$

$2(a + b)$

b. Model the double of $(5x + 3y)$. Are there terms we can combine?



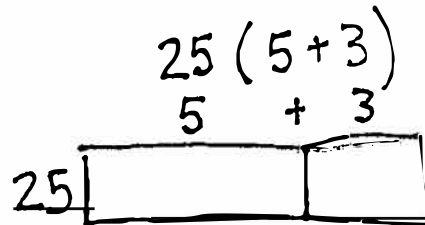
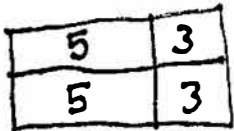
What is an equivalent expression that we can use to represent the double of $(5x + 3y)$?

$10x + 6y$

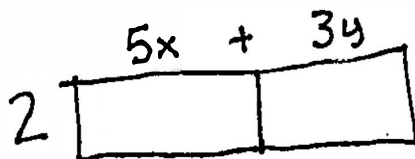
Grade 7

a. Model $2(5 + 3)$ as a rectangular array.

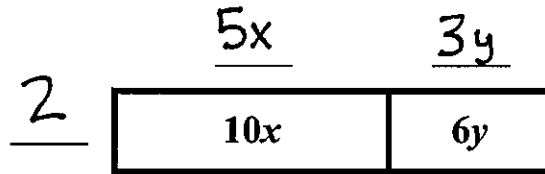
→ **STACKED TAPE DIAGRAM**



b. Model $2(5x + 3y)$ as a rectangular array. Let the variables x and y stand for positive integers.



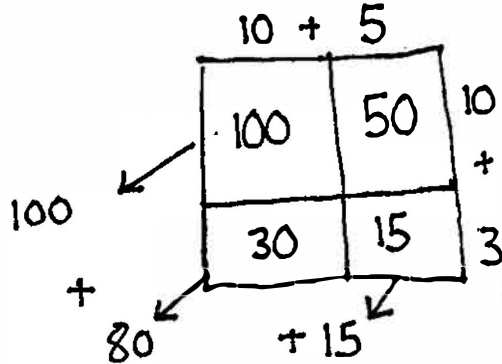
- c. Let the variables x and y stand for positive integers, and let $10x$ and $6y$ represent the area of the two regions in the array in square units. Determine the length and width of each rectangle if the width is the same for each rectangle.



↑ THIS WOULD BE
A HELPFUL SCAFFOLD

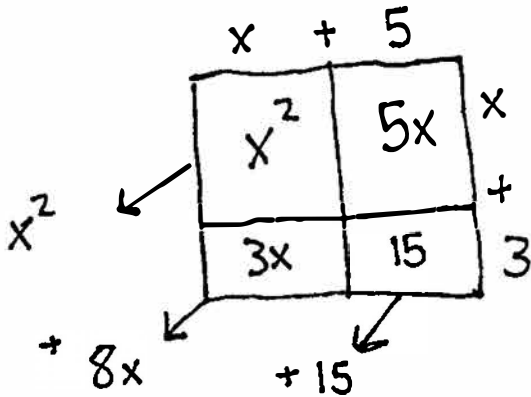
Algebra I

a. In *A Story of Units*, students compute 15×13 as follows:



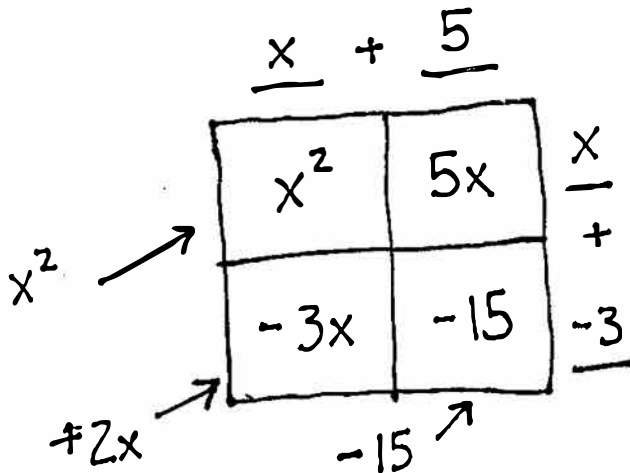
$$100 + 50 + 30 + 15 = 195$$

b. Use the tabular method to multiply $(x + 5)(x + 3) = x^2 + 8x + 15$



TABULAR METHOD:
We can't say "area model" because we can't have a negative area.

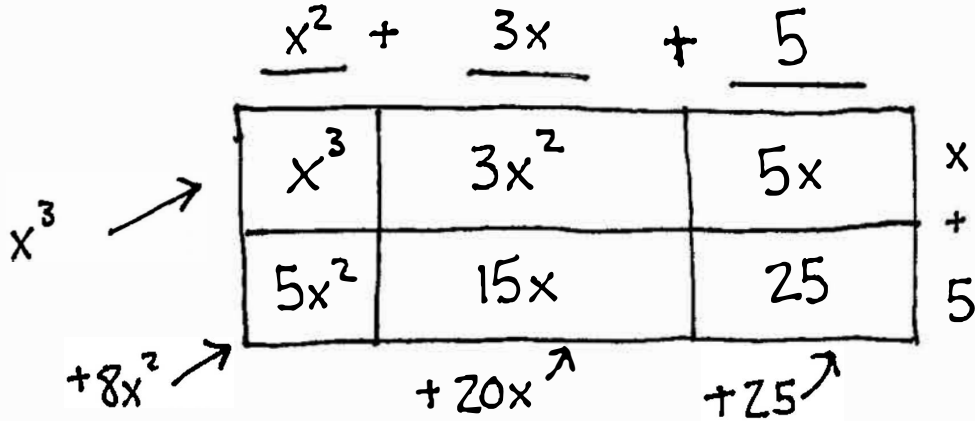
c. Use the reverse tabular method to factor $x^2 + 2x - 15$.



$$x^2 + 2x - 15 = (x + 5)(x - 3)$$

Algebra II

Use the reverse tabular method to calculate $(x^3 + 8x^2 + 20x + 25) \div (x + 5) = x^2 + 3x + 5$



Write down some aha moments you had during this study of coherence.

INTENTIONAL CONNECTIONS ACROSS GRADE LEVELS

What new understandings did you gain?

THE INTENTIONAL PROGRESSION OF UNITS TO
BUILD STUDENTS' CONCEPTUAL UNDERSTANDING
OF MATHEMATICS.

How might the Kindergarten through Precalculus and Advanced Topics coherence of the expression benefit your students?

We can use these models as a
pictorial/representational scaffold for students!

Module Overview Sample Annotations

Directions

1. Write a description of the important information in the Module Overview item.
2. Note whether the Module Overview item provides information about the coherence within the grade level and/or across grade levels. You might use a check or a brief descriptor.

| Item | Description | Coherence | |
|------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|---------------|
| | | Within Grade | Across Grades |
| Table of Contents | <ul style="list-style-type: none"> • MODULE DIVIDED INTO TOPICS • STANDARDS LISTED BY TOPIC • DAYS FOR ASSESSMENT / REMEDIATION • ONE LESSON PER 45-MINUTE DAY • SHOWS LOGICAL PROGRESSION | ✓ | |
| Overview | <ul style="list-style-type: none"> • CLEAR CONNECTIONS ACROSS TOPICS • PROVIDES MORE DETAIL ABOUT EACH TOPIC • OFFERS INFORMATION ABOUT PREVIOUS GRADE LEVEL WORK | ✓ | ✓ |
| Focus Standards | <ul style="list-style-type: none"> • REMIND US OF CONTENT • SHOW EXAMPLES TO HELP UNPACK THE STANDARD • BUILD INSTRUCTIONAL PURPOSE | ✓ | |
| Foundational Standards | <ul style="list-style-type: none"> • SHOW PRECEDING CONTENT • SEE THE ESSENTIAL STANDARDS NEEDED TO ACCESS GRADE LEVEL CONTENT • SUPPORTS AND INFORMS REMEDIATION | | ✓ |

| Item | Description | Coherence | |
|-------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|---------------|
| | | Within Grade | Across Grades |
| Focus Standards for Mathematical Practice | <ul style="list-style-type: none"> • SHOWS MOMENTS WHERE STUDENTS CAN EXHIBIT PRACTICES • ANNOTATED WITHIN LESSONS • INFORMS CONVERSATIONS WITH COLLEAGUES AND ADMINISTRATORS | ✓ | |
| Terminology | <ul style="list-style-type: none"> • PROVIDES DEFINITIONS AND FOCUS • SEPARATED INTO "NEW OR RECENTLY INTRODUCED" AND "FAMILIAR TERMS AND SYMBOLS" • SUPPORTS INSTRUCTIONAL DECISIONS | ✓ | ✓ |
| Suggested Tools and Representations | <ul style="list-style-type: none"> • KNOW WHICH MODELS AND TOOLS WILL BE USED THROUGHOUT THIS MODULE, WHICH SUPPORTS STUDENTS' SUCCESS | ✓ | |
| Assessment Summary | <ul style="list-style-type: none"> • OUTLINES FORMAT AND WHEN STANDARDS ARE ASSESSED • IMPORTANT IN PLANNING AND PREPARATION | ✓ | |

Lesson Demonstration Sample Annotations

Directions

1. As the facilitator demonstrates G7 M3 L16, follow along by using the student pages in the Virtual Engagement Materials, Pages 27-33.
2. As the facilitator models the lesson, think about the following: How are the components of rigor embedded in the lesson?

Conceptual Understanding:

- USING A COMPASS TO DEVELOP THE DEFINITION OF A CIRCLE
- DERIVING THE FORMULA FOR CIRCUMFERENCE BY COUNTING DIAMETERS AND DISCOVERING THE RATIO OF THE CIRCUMFERENCE TO THE DIAMETER OF THE CIRCLE

Procedural Skills and Fluency:

- USING A COMPASS AND RULER
- PERFORMING OPERATIONS WITH DECIMALS AND FRACTIONS
- ROUNDING TO THE NEAREST TENTH OR HUNDREDTH

Application:

- APPLYING KNOWLEDGE OF RADIUS, DIAMETER, AND CIRCUMFERENCE BY FINDING THE PERIMETER OF SEMICIRCLES AND OTHER SHAPES

Search and Find Sample Annotations

Directions

1. Look in the body of the Lesson Demonstration Teacher Edition
2. Record the page number of each item you find for reference when we debrief.
3. Save the last column for taking notes as we debrief each component as a group.

| Look for | Page Number | Notes |
|-----------------------------------------------|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Student Outcomes | 36 | <ul style="list-style-type: none"> • DRIVE CONTENT OF LESSON • SERVES AS THE "COMPASS" for strategic and INFORMED INSTRUCTIONAL DECISIONS |
| Lesson with a Lesson Note | 36 | <ul style="list-style-type: none"> • TEACHING TIPS AND CONNECTION TO CONTENT AT OTHER GRADE LEVELS • DRAWS ATTENTION TO ANYTHING THAT TEACHERS MAY NEED TO PREPARE |
| Example Followed by an Exercise | 36 - 40 | <ul style="list-style-type: none"> • EXAMPLES: Typically, TEACHER - DRIVEN • EXERCISES: Typically, STUDENT - DRIVEN • MAY INCLUDE TIME SUGGESTIONS |
| Closed Bullet Followed by an Open Bullet | 37 | <ul style="list-style-type: none"> • CLOSED: SUGGESTED TEACHER DIALOGUE • OPEN: POSSIBLE STUDENT RESPONSE • THESE ARE SUGGESTIONS. TEACHERS ARE ENCOURAGED TO CUSTOMIZE. |
| Reference to a Mathematical Practice Standard | 37 | <ul style="list-style-type: none"> • ANNOTATED ON LEFT • INDICATED WITH BRACKETS. STRONG OPPORTUNITY TO ENGAGE STUDENTS IN THE MATHEMATICAL PRACTICE |

| Look for | Page Number | Notes |
|------------------------------|--------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Box around an Item | 36-37 39-41 43-46 | <ul style="list-style-type: none"> • SHOWS WHAT APPEARS IN THE STUDENT EDITION • ANSWERS NOT INCLUDED FOR STUDENTS • IF AN ITEM IS NOT BOXED, IT DOES NOT APPEAR IN THE STUDENT EDITION |
| Scaffolding Box | SEE: ALGEBRA II MODULE 1 LESSON 9 | <ul style="list-style-type: none"> • STRATEGICALLY PLACED IN LESSON'S MARGIN NOTES • ELABORATES ON THE USE OF SPECIFIC SCAFFOLDS TO MEET THE NEEDS OF VARIOUS LEARNERS |
| Closing | 40-41 | <ul style="list-style-type: none"> • NEARLY ALL LESSONS END WITH A CLOSING. SOME TWO DAY LESSONS DO NOT • TIES UP THE LESSON WITH A "NICE, NEAT BOW" |
| Lesson Summary | SEE: ALGEBRA 1 MODULE 1 LESSON 5 | <ul style="list-style-type: none"> • CONTAINS KEY INFORMATION OR DEFINITIONS FROM THE LESSON • EXIT TICKET FOLLOWS THE LESSON SUMMARY |
| Blank Exit Ticket | 42 | <ul style="list-style-type: none"> • CAN BE USED BY TEACHER TO: <ul style="list-style-type: none"> - DO THE MATH - MAKE COPIES |
| Exit Ticket Sample Solution | 43 | <ul style="list-style-type: none"> • WRITTEN AS ONE POSSIBLE RESPONSE • ENCOURAGE FLEXIBLE THINKING • EXIT TICKET DATA INTENDED TO INFORM UPCOMING INSTRUCTION |
| Problem Set Sample Solutions | 44 | <ul style="list-style-type: none"> • WRITTEN AS ONE POSSIBLE RESPONSE • "FLEX BANK OF PROBLEMS" TO SUPPORT AND REINFORCE INSTRUCTION • NOT BUILT INTO 45-MINUTE LESSON |

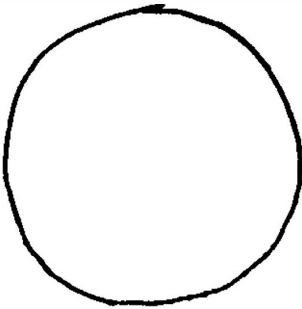
Sample Work and Annotated Complexities

Lesson 16: The Most Famous Ratio of All

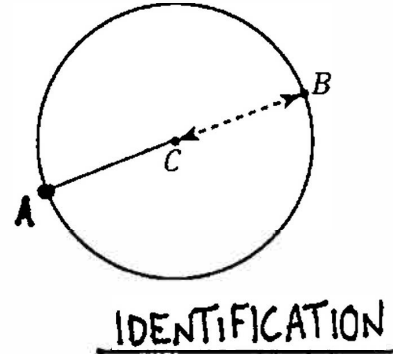
Classwork

Opening Exercise

- a. Using a compass, draw a circle like the picture to the right.



COMPLEXITIES
 - USING A COMPASS
 - CLARITY OF DRAWING "LIKE" CIRCLE



C is the center of the circle.
 The distance between C and B is the radius of the circle.

- b. Write your own definition for the term circle.

A SET OF ALL POINTS
 Equal DISTANCE FROM
 A GIVEN CENTER

| <u>COMPLEXITIES</u> | <u>IDENTIFICATION</u> |
|--------------------------------------------------------------------|-----------------------|
| - NEW TERMS - CONNECTION TO COMPASS - WRITING OWN DEFINITION | |

- c. Extend segment CB to a segment AB in part (a), where A is also a point on the circle.

SEE CIRCLE ABOVE

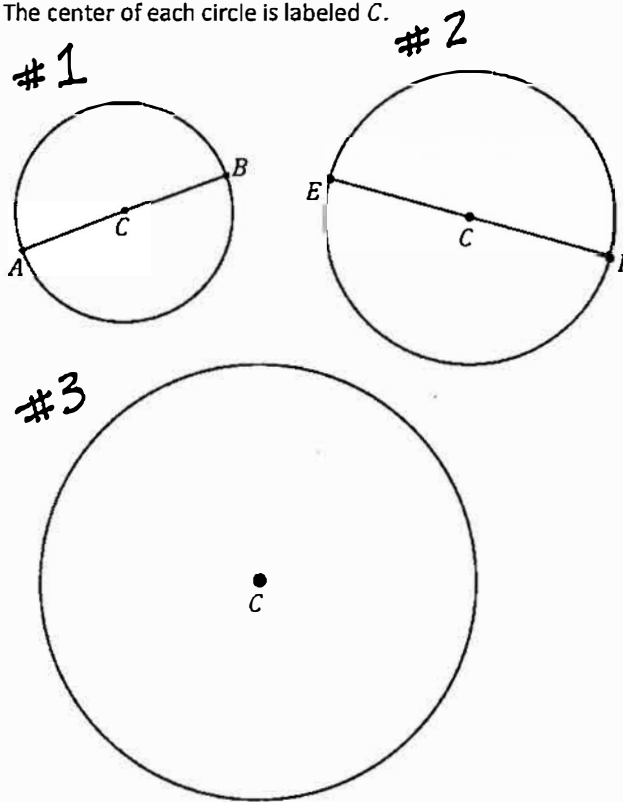
The length of the segment AB is called the diameter of the circle.

| <u>COMPLEXITIES</u> | <u>IDENTIFICATION</u> |
|----------------------------------------------------------|-----------------------|
| C: - LANGUAGE OF EXTEND - DIRECTION TO EXTEND SEGMENT | C: |
| D: - ABSTRACTION - REQUIRES LANGUAGE CONNECTIONS | D: |

- d. The diameter is TWICE as long as the radius.

e. Measure the radius and diameter of each circle. The center of each circle is labeled C.

$CB = 1.5 \text{ cm}$ $CF = 2 \text{ cm}$
 $AB = 3 \text{ cm}$ $EF = 4 \text{ cm}$
 RADIUS (#3) = 3 cm
 DIAMETER (#3) = 6 cm



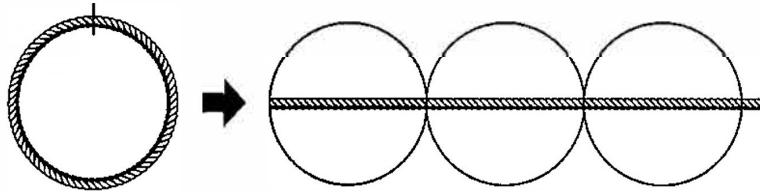
| COMPLEXITIES | IDENTIFICATION |
|--------------------------------|----------------|
| - MEASURE USING RULER | # 1: |
| - INCLUSION OF UNIT OF MEASURE | # 2: |
| - REMOVAL OF LABELS (# 3) | # 3: |

f. Draw a circle of radius 6 cm.

| COMPLEXITIES | IDENTIFICATION |
|----------------------------|----------------|
| - USING TWO TOOLS TOGETHER | |
| - FITTING CIRCLE IN SPACE | |

Mathematical Modeling Exercise

The ratio of the circumference to its diameter is always the same for any circle. The value of this ratio, $\frac{\text{Circumference}}{\text{Diameter}}$, is called the number π and is represented by the symbol π .



DURING THIS EXERCISE, COUNT BY DIAMETERS

Since the circumference is a little greater than 3 times the diameter, π is a number that is a little greater than 3. Use the symbol π to represent this special number. Pi is a non-terminating, non-repeating decimal, and mathematicians use the symbol π or approximate representations as more convenient ways to represent pi.

- $\pi \approx 3.14$ or $\frac{22}{7}$.
- The ratios of the circumference to the diameter and $\pi : 1$ are equal.
- Circumference of a Circle = $\pi \times$ Diameter.

Example

- a. The following circles are not drawn to scale. Find the circumference of each circle. (Use $\frac{22}{7}$ as an approximation for π .)

#1

$C \approx 66 \text{ cm}$

#2

$C \approx 286 \text{ ft}$

#3

$C \approx 110 \text{ m}$

COMPLEXITIES

- COMPUTATION WITH FRACTIONS
- NON-SCALE MODELS
- GIVEN RADIUS, RATHER THAN DIAMETER (#3)

IDENTIFICATIONS: _____

- b. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest tenth. (Use 3.14 as an approximation for π .)

$C \approx 73.5 \text{ cm}$

| COMPLEXITIES | IDENTIFICATION |
|--------------------------------------------------------------------------------------------------------------------------------------------|-----------------------|
| <ul style="list-style-type: none"> - DECIMAL APPROXIMATE - ROUNDING TO NEAREST TENTH - NO VISUAL SCAFFOLD | |

- c. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest hundredth. (Use the π button on your calculator as an approximation for π .)

$$C \approx 73.51 \text{ cm}$$

| COMPLEXITIES | IDENTIFICATION |
|--------------------------------------------------------------------------------------------------------------------------------------|----------------|
| <ul style="list-style-type: none"> - USING THE π BUTTON ON CALCULATOR - ROUNDING TO HUNDREDTH | |

- d. A circle has a radius of r cm and a circumference of C cm. Write a formula that expresses the value of C in terms of r and π .

$$C = \pi \times d$$

$$C = \pi \times 2 \times r$$

| COMPLEXITIES | IDENTIFICATION |
|----------------------------------------------------------------------------------------------------------------------------|----------------|
| <ul style="list-style-type: none"> - ABSTRACTION WITH USE OF VARIABLES - LANGUAGE OF "IN TERMS OF" | |

- e. The figure below is in the shape of a semicircle. A semicircle is an arc that is half of a circle. Find the perimeter of the shape. (Use 3.14 for π .)

$$C = \pi \times d$$

$$\frac{1}{2}C = \frac{1}{2} \times \pi \times d$$

$$\frac{1}{2}C = \frac{1}{2} \times \pi \times 8$$

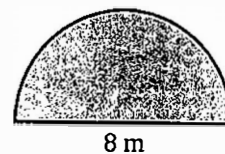
$$\frac{1}{2}C = \pi \times 4$$

$$\frac{1}{2}C \approx 12.56$$

$$P = \frac{1}{2}C + d$$

$$P \approx 12.56 + 8$$

$$P \approx 20.56 \text{ m}$$



THE PERIMETER OF THE SEMICIRCLE IS APPROXIMATELY 20.56 M.

| COMPLEXITIES | IDENTIFICATION |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|
| <ul style="list-style-type: none"> - USE OF SEMI-CIRCLE - NEW LANGUAGE (ARC, PERIMETER) - ADDING IN 8 M AS PART OF PERIMETER | |

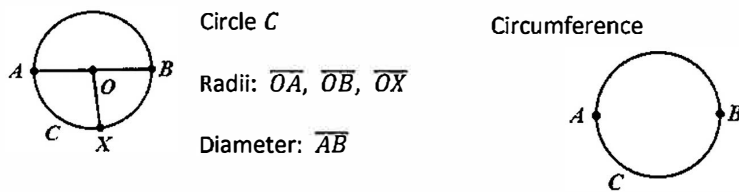
Relevant Vocabulary

CIRCLE: Given a point O in the plane and a number $r > 0$, the *circle with center O and radius r* is the set of all points in the plane whose distance from the point O is equal to r .

RADIUS OF A CIRCLE: The *radius* is the length of any segment whose endpoints are the center of a circle and a point that lies on the circle.

DIAMETER OF A CIRCLE: The *diameter of a circle* is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If r is the *radius* of a circle, then the diameter is $2r$.

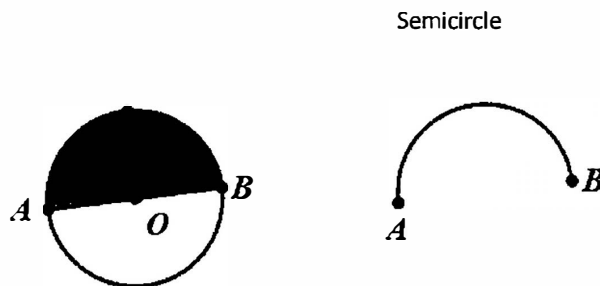
The word *diameter* can also mean the segment itself. Context determines how the term is being used: *The diameter* usually refers to the length of the segment, while *a diameter* usually refers to a segment. Similarly, *a radius* can refer to a segment from the center of a circle to a point on the circle.



CIRCUMFERENCE: The circumference of a circle is the distance around a circle.

PI: The number *pi*, denoted by π , is the value of the ratio given by the circumference to the diameter, that is $\pi = \frac{\text{circumference}}{\text{diameter}}$. The most commonly used approximations for π is 3.14 or $\frac{22}{7}$.

SEMICIRCLE: Let C be a circle with center O , and let A and B be the endpoints of a diameter. A *semicircle* is the set containing A , B , and all points that lie in a given half-plane determined by \overline{AB} (diameter) that lie on circle C .



CLOSING: "IN YOUR OWN WORDS, DESCRIBE WHAT YOU LEARNED TODAY."

TRY TO INCLUDE: CIRCLE, RADIUS, DIAMETER, CIRCUMFERENCE

Sample Annotated Lesson Preparation Template

| | | |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| Lesson Number: 16 Title: THE MOST FAMOUS RATIO OF ALL | | Day/Date: |
| Lesson Type (circle one): <input checked="" type="checkbox"/> Problem Set <input type="checkbox"/> Exploration <input type="checkbox"/> Socratic <input checked="" type="checkbox"/> Modeling Cycle | | |
| Student Outcomes | <ul style="list-style-type: none"> • STUDENTS DEVELOP THE DEFINITION OF A CIRCLE USING DIAMETER AND RADIUS • STUDENTS KNOW THAT THE DISTANCE AROUND A CIRCLE IS CALLED THE CIRCUMFERENCE AND DISCOVER THAT THE RATIO OF THE CIRCUMFERENCE TO THE DIAMETER OF A CIRCLE IS A SPECIAL NUMBER CALLED PI, WRITTEN π • STUDENTS KNOW THE FORMULA FOR CIRCUMFERENCE C OF A CIRCLE OF DIAMETER d AND RADIUS r. THEY USE SCALE MODELS TO DERIVE THESE FORMULAS • STUDENTS USE $\frac{22}{7}$ AND 3.14 AS ESTIMATES FOR π AND INFORMALLY SHOW THAT π IS SLIGHTLY GREATER THAN 3. | |
| Materials | <ul style="list-style-type: none"> • COMPASS • RULER • CALCULATOR | |
| Fluency (as needed) | <p>THROUGHOUT THE WEEKS LEADING TO THIS LESSON, STUDENTS HAVE ENGAGED IN RAPID WHITEBOARD EXCHANGES EMPHASIZING FLUENCY WITH:</p> <ul style="list-style-type: none"> – USING A COMPASS – USING A RULER WITH THE UNIT OF CENTIMETERS – MULTIPLICATION WITH FRACTIONS AND DECIMALS | |
| Classwork | <p>OPENING EXERCISE</p> <p>a) MD</p> <p>b) MD</p> <p>c) MD</p> <p>d) MD</p> <p>e) MD, CD, CD</p> <p>f) E</p> | <p>MATHEMATICAL MODELING : MD EXERCISE</p> <p>EXAMPLE</p> <p>a) MD, CD, MD</p> <p>b) MD</p> <p>c) CD</p> <p>d) MD</p> <p>e) MD</p> |
| Key | <ul style="list-style-type: none"> ▪ Must Do (M) ▪ Could Do (C) ▪ Extension Problems (E) | |

