EUREKA MATH®

User Guide

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Introduction

Eureka Math® is a comprehensive mathematics curriculum designed to help students build a deep and lasting understanding of mathematical concepts. Rooted in a coherent progression of content from Prekindergarten through high school, it equips teachers with robust instructional tools and resources to inspire confidence and curiosity in every learner.

Eureka Math lessons emphasize conceptual understanding, fluency, and application, supported by clear guidance for fostering mathematical discourse and engaging students in reasoning, problem-solving, and making connections across topics. With embedded supports for differentiation, and purposeful integration of visual models, manipulatives, and discussion routines, *Eureka Math* helps educators create classrooms where students actively construct knowledge and develop strong mathematical identities.

Visit the digital platform to view or print teacher and student materials, including homework support, or to access daily lesson videos, digital tools, and other teaching supports. Customize assessments on the platform and use the platform's tools to track student progress and inform next steps.

This guide contains detailed resources about recommended instructional routines and discussion-based assessment questions to improve student engagement and to integrate language, social, and emotional development into instruction.

By combining rigor, clarity, and coherence, *Eureka Math* prepares students not just for the next lesson or test, but for a lifetime of mathematical thinking and problem-solving.

Thank you for the work you do every day. This guide is meant to stand beside you as you support meaningful discourse and build the kind of classroom where ideas are tested, refined, and shared. We're proud to partner with you to help students grow in confidence and understanding.

Cristina Metcalf, Senior Director—Mathematics

Elements of *Eureka Math*

The digital platform presents an enhanced version of the original *Eureka Math* curriculum, including a variety of instructional supports and resources. Additional information about instructional routines and discussion-based assessment questions is also provided in this guide.

While this guide focuses on these additional supports and resources, the instructional foundation continues to be the *Eureka Math* curriculum, which includes the following components:

- Teacher Edition (TE) (print and digital) including
  - foundational/previous grade standard knowledge needed for each module,
  - focus standards addressed for each module and topic, and
  - coherence links to previous and future grade modules to promote connection-building.
- Student *Learn, Practice, and Succeed* books (Grades K–8) and Student Edition (SE; PK and Grades 9–12) (print and digital)
- Assessment materials (print and PDF; digital versions available)

The digital version of *Eureka Math* along with the information in this guide provide a variety of additional features to enhance both the teacher and student experience.

Built-in digital assessments also provide educators with

- access to a database of formative items created by the curriculum's teacher–writers and
- analytics tools to help teachers track student progress and identify areas of need.

Key Elements	How Does This Element Help Teachers?
Digital Assessments for Grades 1–12 • Topic-level quizzes • Mid-Module and End-of-Module Assessments	Digital assessments allow teachers to implement topic-level quizzes. Teachers can also benefit from the bank of Mid-Module and End-of-Module Assessment questions.
Data Reporting • Automatic scoring in the digital platform • Customizable score release • Reporting by student, class, test, question, and standard	Data reporting and automatic scoring streamline the assessment and grading processes. Reports assist teachers in identifying when a review of *Eureka Math* lessons, including video lessons, may be beneficial.
Customizable Assessments • Access to all grades • Questions tagged by grade, module, topic, and standard • Ability to schedule assessment launch	Teachers can customize and modify assessments to reflect *Eureka Math* learning and to work with learning structures. Assessments can be flexibly scheduled to accommodate any class.

Key Elements	How Does This Element Help Teachers?
Daily **video lessons** created by a Great Minds teacher	Video lessons provide content-focused, knowledge-building instruction and meet the varied needs of educators: • **In Person:** Educators can teach in accordance with the TE (print or digital) and leverage video lessons when students need additional support or to differentiate instruction (e.g., using a lesson video as one of a set of stations). • **Blended:** Teachers may assign video lessons before or after they facilitate instruction. • **Preparation:** Teachers may use the videos as an additional tool in their lesson preparation. Introductory videos are included with Module 1 to help teachers and their students get to know the Great Minds teachers.
A **Topic Facilitation** slide deck for every topic	Lesson-level slides support facilitation of the Application Problem (Grades PK–5) and provide an opportunity to debrief the lesson with students (Grades PK–12). Topic Discourse slides support facilitation of discussion across lessons and can be useful when a daily discussion or debrief is not possible. Recommended instructional routines are provided in the Topic Discourse slides to support engagement and integrate language, social, and emotional development into academic instruction. (See section on Instructional Routines under Opportunities for Teacher-Facilitated Discourse in *Eureka Math* and Appendix B for more information on suggested routines.)

Key Elements	How Does This Element Help Teachers?
Discussion-based assessment questions for Mid-Module and End-of-Module Assessment tasks	Discussion-based assessment questions provide educators with a structure for digging into student thinking about the essential knowledge students have learned. The questions also equip educators with information about student learning that can be used to make next-step instructional decisions. (See section on Discussion-Based Assessment and Appendix A for more information.)
The ***Teach Eureka*** **video series** with teacher-facing content support for each grade and module	This series contains 18 one-hour sessions for each grade (PK–12), organized sequentially by module. These videos explain the mathematical concepts and instructional strategies necessary for teachers to make *Eureka Math* their own.

Enhanced Learning Through Discourse

Opportunities for Teacher-Facilitated Discourse in *Eureka Math*

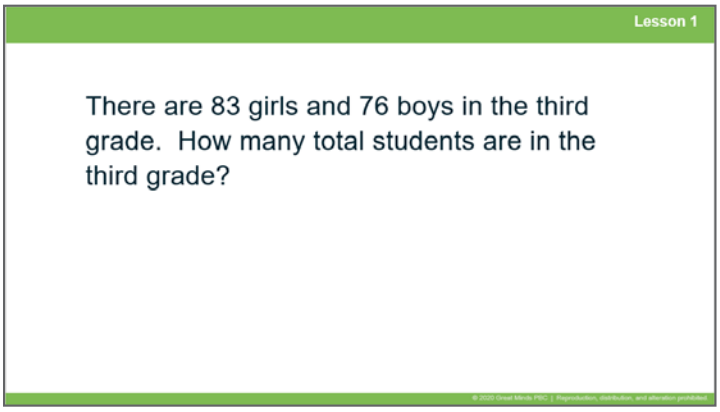
Discourse is an essential component of effective math instruction. Through discourse, students have opportunities to share ideas, clarify understandings, construct arguments about how and why things work, develop precise language skills, and learn to view problems from diverse perspectives. Discourse, therefore, is a powerful tool that teachers can use to advance the mathematical learning of the whole class (NCTM 2014).

Eureka Math provides teachers with Topic Facilitation materials to incorporate discussion throughout the learning process. The Topic Facilitation materials have two core components:

- Lesson slides (Grades PK–5 only) and Student Debrief slides (Grades PK–12)
- Topic Discourse slides

Lesson Slides (PK–5)

Lesson slides allow teachers to project the Application Problem to engage students in discussions around problem-solving strategies and student sensemaking throughout each lesson. Where applicable, sample student work is provided to engage students in comparing, explaining, or building on math ideas.



Example Application Problem (G3 M1 L1) with notes to the teacher to guide facilitation

Student Debrief Slides (PK–12)

Student Debrief slides provide visual companions as needed to support a meaningful summarization of each lesson. They can be used as an opportunity to generate discussion and further students' mathematical reasoning and critical thinking.

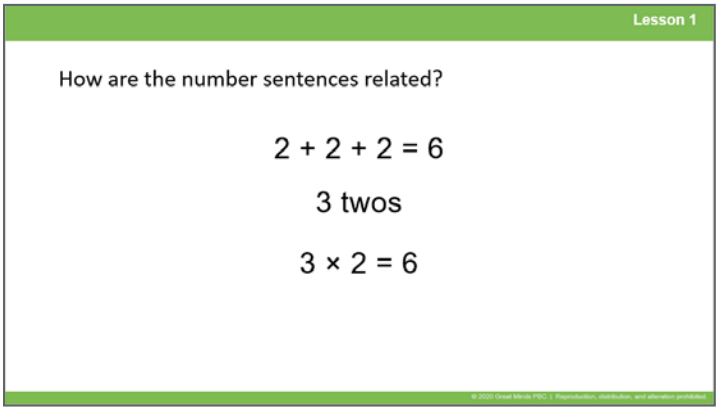
There are four approaches to facilitating discussion in a Student Debrief: Sample Solution, Error Analysis, Connection, and Impact. The approach for any given lesson is indicated in the notes of the Student Debrief slide for that lesson along with accompanying suggestions for facilitation.

Sample Solution: Analyze samples of accurate solutions (student- or teacher-created) to reinforce procedures, strategies, and pictorial representations.

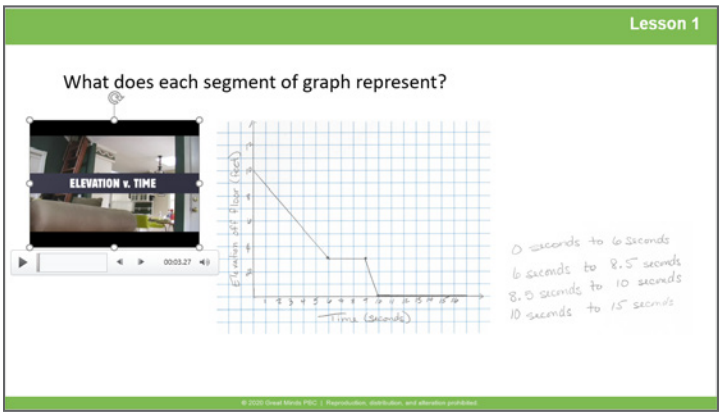
Error Analysis: Identify and correct errors and misconceptions in sample solutions.

Connection: Recognize a connection or relationship across problems or key concepts within a lesson or from previous learning.

Impact: Observe and understand the impact of a key concept within the lesson (often paired with another approach to provide a visual support).



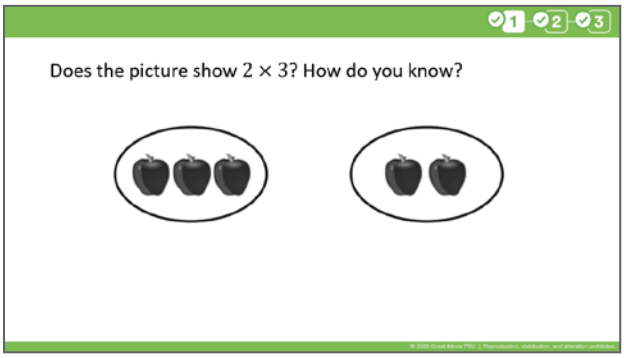
Example Student Debrief (G3 M1 L1) with a question to elicit student thinking, an image of the problems, and a note on the recommended discussion approach



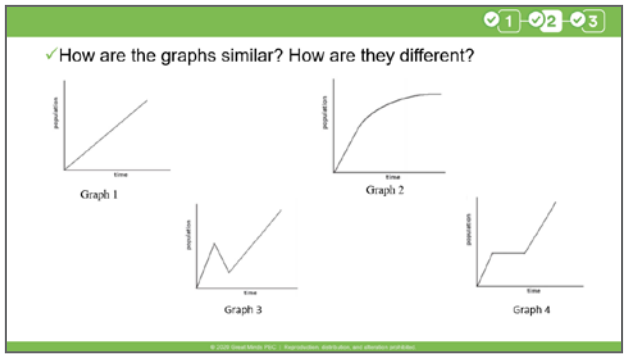
Example Student Debrief (Alg1 M1 L1) with a question relating the sections of a piecewise linear graph to a video

Topic Discourse Slides

In addition to making discourse a regular part of the daily lesson experience, *Eureka Math* also includes materials for topic-level discourse. Topic Discourse slides contain specific tasks and sample student work from the lessons in the topic and are intended to spark thinking and discussion beyond the learning objectives of each day's lesson. The features of these checkpoints, as highlighted in the image below, are consistent from Prekindergarten through Grade 12. Since *Eureka Math* lessons gradually build in complexity, Topic Discourse materials engage students in generalizing about and reflecting on the essential knowledge they have been learning across a series of lessons.



Example Grade 3



Example Algebra 1

Routines That Promote Language and Content Development

When preparing to deliver *Eureka Math* lessons, it is important to remember that we are not solely building content knowledge. The whole child must be considered during the preparation process, which should include the incorporation of instructional routines that work toward language, social, and emotional goals in addition to academic goals. Instructional routines also help increase access, engagement, and equitable contributions throughout the discussion while maintaining high levels of student thinking.

The *Eureka Math* teacher–writers recommend that teachers consider incorporating the instructional routines listed below into their daily lesson delivery. Suggested routines can be found in the Topic Facilitation slide decks, but teachers are encouraged to choose or adapt routines based on the immediate needs of their students. More detailed information can be found in Appendix B.

Instructional Routines

Math Chat creates an open-ended space for sharing mental math strategies and developing number sense, flexibility, efficiency, and accuracy.

Always, Sometimes, Never promotes sense-making and mathematical discussion as students support a claim with examples and nonexamples.

Co-Construction provides structured support for contextualizing and decontextualizing problems, which helps students build abstract reasoning.

Which One Doesn't Belong? promotes metacognition and mathematical discourse as students use precise language to compare different representations or examples.

Numbered Heads helps groups function effectively by encouraging students to build consensus and holds each student accountable for learning the material.

Critique a Flawed Response promotes effective communication techniques for critiquing others' work, correcting errors, and clarifying meaning.

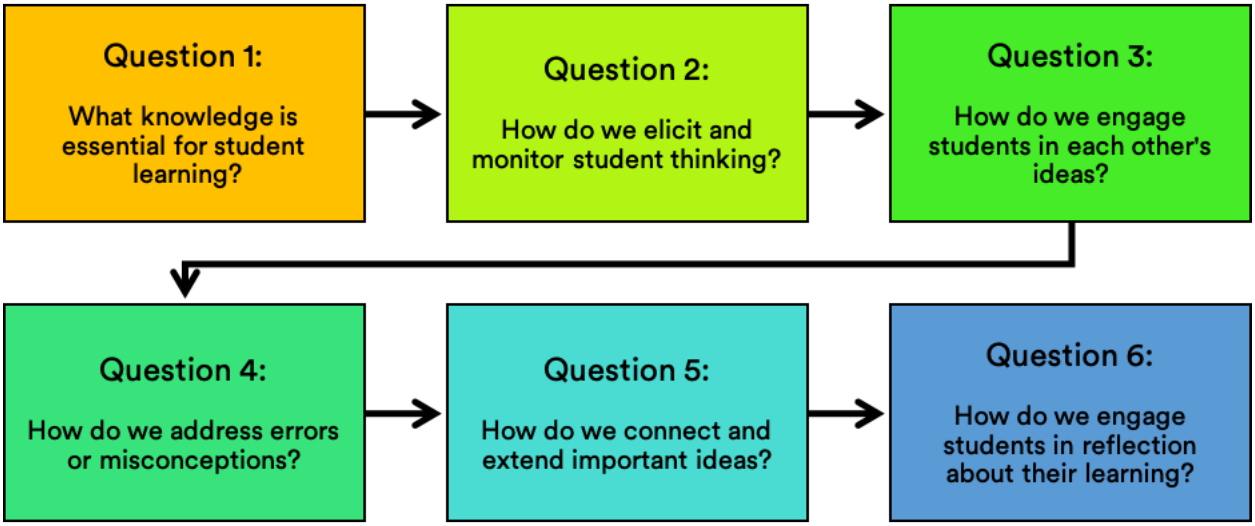
Take a Stand supports students in making arguments and critiquing the reasoning of others.

Five Framing Questions supports students in analyzing a work sample or solution strategy by providing them with questions to frame their thinking.

Stronger and Clearer Each Time provides a structured, interactive opportunity for students to revise and refine their written language through rehearsal.

Effective Math Discourse—Six Guiding Questions

As math education research has progressed, math educators and researchers have published a variety of frameworks for teachers to use to prepare for and facilitate effective math discourse. The six questions explored below are a useful synopsis of these frameworks. Additional information about these guiding questions can be found in Appendix A.



Driving Question	Teacher Actions
What knowledge is essential for student learning?	• Identifies the core learning outcomes for the module, topic, and lessons • Identifies how essential knowledge builds from prior learning • Selects tasks for instruction and practice that bridge prior learning to new knowledge
How do we elicit and monitor student thinking?	• Promotes reasoning and sensemaking throughout instruction • Asks questions such as, "How do you know that?" or "Why did you decide to...?" • Provides ample opportunities for students to hear and use key mathematical language with precision as they work

Driving Question	Teacher Actions
How do we engage students with each other's ideas?	• Provides opportunities for students to share their work with diverse partners, pose questions to each other, and build common understandings • Supports students in using mathematical language to communicate their reasoning and critique the arguments of others • Engages students in discussion about a sample of student work, a comparison of related problems or work samples, errors in reasoning, or the impact of an aspect of a task in a problem
How do we address errors or misconceptions?	• Allows students time to grapple with challenging work collaboratively or independently • Celebrates mistakes as opportunities to learn • Supports productive struggle by providing constructive feedback and asking questions to probe thinking, make the mathematics visible, or invite reflection • Keeps error analysis anonymous to focus attention on the thinking, not the author
How do we connect and extend important math ideas?	• Consistently asks students to look for and make use of connections among representations, solution methods, and essential knowledge • Encourages students to look for ways to make their work more efficient or clearer to understand • Focuses students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation
How do we engage students in reflection about their learning?	• Selects and sequences student work samples to illuminate specific knowledge students should gain from the lesson • Highlights student representations, methods, and language for analysis and comparison to the essential learning outcomes • Uses evidence of student knowledge to inform next-step instructional decisions

Adapted from Principles to Actions (NCTM 2014), 5 Practices for Orchestrating Productive Math Discussions (Smith and Stein 2018), Children's Mathematics: Cognitively Guided Instruction (Carpenter et al. 2015), and "Principles for the Design of Mathematics Curricula: Promoting Language and Content Development" (Zwiers et al. 2017).

Discussion-Based Assessment

Assessment should support learning by providing teachers and students with useful information. The National Council of Teachers of Mathematics defines assessment as "the process of gathering evidence about a student's knowledge of, ability to use, and disposition toward mathematics and of making inferences from that evidence for a variety of purposes" (NCTM 1995, 3). The council further defines four distinct purposes for assessment of school mathematics:

- Monitoring students' progress
- Making next-step instructional decisions
- Evaluating students' achievement
- Evaluating program structures and teaching practices

The resources in *Eureka Math* already achieve these goals because they contain rigorous tasks that require students to demonstrate their knowledge at strategic points over the course of a module. When used formatively, the information gleaned from student work on assessment tasks can guide decisions and support continual growth toward essential mathematics goals.

The resources provided here expand the capacity of classroom teachers to assess students' knowledge and growth through discussion-based assessments. A discussion-based assessment is a one-on-one conversation between the teacher and a student about what the student has learned at each module milestone. They are opportunities to connect with students at a deeper level as the teacher digs into students' understanding about the essential learning of the module.

After students have completed and submitted a given module assessment, teachers follow up with students individually by choosing a question for discussion-based assessment.

Posing Clarifying Questions to Assess Knowledge

Each discussion-based assessment question set begins with asking students how they solved a particular problem from the assessment. As teachers listen to the student's explanation of the submitted written work, they look for key indicators (discerned through their study of the module and lesson materials) that the student understands the essential knowledge the module aims to impart.

Sometimes, students can correctly solve a problem but still have misconceptions about the underlying mathematics. That is why the questioning used during the assessment interview must go beyond whether the solution is correct or not. Once a student has answered the initial

question asking them to explain their thought process, the teacher might follow up with one or two clarifying questions focused on key features of the work. These questions are not intended to teach the student or to guide them toward a correct solution but rather to elicit elaboration about a specific part of the completed work or to help them clarify their thinking about the mathematical structures in the task.

Recommendations for follow-up clarifying questions are included in the directions for conducting discussion-based assessment found below. Teachers should use these recommendations to help identify the essential knowledge that should be salient from work during the module. They might also adapt the recommended question sets to elicit student thinking at any point during the module, including during Problem Set or Exit Ticket work. Monitoring evidence of student thinking can occur continuously to inform each instructional step. The evidence gathered can identify and thus help address potential gaps in knowledge or misconceptions before that reasoning becomes more solidified and difficult to remediate.

Directions for Conducting Discussion-Based Assessment (PK–12)

Step 1: Assign the Mid-Module or End-of-Module Assessment Tasks.

Step 2: Review student responses. Identify strengths in student knowledge, and note any errors that may indicate possible misconceptions.

Step 3: Schedule time to meet each student for individual discussion about a problem you have selected based on your analysis of student responses from Step 2.

Step 4: Conduct the discussion.

a) Begin by having the student review their solution to the selected problem. Ask the student to respond to one of the initial prompts shown in the table below.

b) While the student explains their thinking, record evidence indicating student understanding or misconceptions about the essential knowledge in the problem.

c) If needed, pose follow-up prompts to clarify student thinking and continue to record evidence. Recommendations for clarifying questions can be found in the table.

Step 5: Analyze data gathered from student interviews for common areas of need. Use the data as you plan for upcoming instruction.

Initial Prompts (Choose 1.)	**Follow-Up Prompts** (Use as needed.)
Tell me how you solved this problem, Explain the reasoning you used as you solved this problem. How did you find your answer? Talk to me about your solution. Walk me through the steps you took to solve this problem.	Tell me more about.... Can you explain that in a different way? How does _____ relate to _____? What did you mean when you said...? How would you check your answer? How did you go from _____ to _____? Why did you decide to...?

Taking Action to Continue Knowledge Building

Mid-Module and End-of-Module Assessment tasks are not intended to be a summative snapshot of where students are in their learning. Rather, the tasks included in these resources are avenues for eliciting and using student thinking to make all-important decisions about how best to meet students' needs in their progression toward mastery. The power of effective instruction and assessment is in making decisions to meet students' needs while honoring the essential mathematics.

Additional Resources

Eureka Math with *Eureka Math Equip*®

Eureka Math Equip includes module-by-module preassessments that identify gaps in content knowledge and link seamlessly to targeted instruction, setting students up for success in the module ahead. The following components are available for all modules in *Eureka Math* at Grades 1–12.

- **Pre-Module Assessments:** These adaptive assessments use branch logic to create a custom question path for each student to help teachers pinpoint conceptual gaps and misunderstandings before every module.
- **Assessment Reporting:** Teachers can see individual student performance as well as class trends and receive recommendations for grouping students with similar needs on supporting concepts.
- **Instructional Recommendations:** Supporting activities, lessons, or lesson components draw on previous grade-level content to help students overcome gaps in foundational knowledge identified in the pre-module assessments.
- **Streamlined Pacing:** Versatile pacing suggestions help teachers keep students on track with current grade-level work while catching up on previous material—without adding instructional days.

Eureka Math Equip Feature	Using *Eureka Math* on the Digital Platform
Pre-Module Assessments • Identify gaps and misconceptions in foundational knowledge	• *Eureka Math* lesson videos are available for previous grade levels. • Assign *Eureka Math* video lessons from previous grades to help reinforce skills.
Assessment Reporting • Foundational Knowledge Checklist for manual entry • Automatic scoring in the digital platform	• Recommended grouping assists with using *Eureka Math* video lessons in small group instruction or as additional support. • Identify students who could benefit from *Eureka Math* video lessons outside of the classroom.
Instructional Recommendations • Supporting lessons (identified individually) • Supporting lesson components (single document) • Supporting fluencies (single document)	• Supporting lessons may be reinforced by students independently viewing *Eureka Math* video lessons. • *Eureka Math* lessons are previewed and reviewed on an as-needed basis for students.

***Eureka Math Equip* Feature**	**Using *Eureka Math* on the Digital Platform**
Streamlined Pacing • Addresses learning gaps while staying on track • Options for lesson consolidations and omissions	• Streamlined pacing may be supported by students independently viewing *Eureka Math* video lessons. • Consider how *Eureka Math* video lessons can aid in consolidation. • Use *Eureka Math* to support pacing decisions.

Response to Intervention with *Math Catalyst*

Math Catalyst, the newest resource in *Eureka Math*® Assessments, provides educators with the content they need to supplement their core curriculum with practice, application, and Mini Lessons that support students at every tier of learning.

Math Catalyst consists of seven strands across Kindergarten through Grade 5.

- Early Numeracy
- Fractions as Numbers
- Place Value
- Addition
- Subtraction
- Multiplication
- Division

Each strand is composed of a progression of Concepts, and each Concept includes a series of components that can be used in small group direct instruction as well as stations.

Eureka Math with *Eureka Math* Professional Development

Eureka Math offers a variety of professional development resources to support growth in teaching and learning. Consult https://greatminds.org/professional-learning for the latest available options.

Professional Development	Using with *Eureka Math*
Video Library and Webinars • On-demand short videos and longer webinars • Various *Eureka Math* implementation strategies and practices for grades PK–12	• Learn about the basics of *Eureka Math*. • Review to assist with preparing to teach a lesson. • Recognize strategies and practices used throughout *Eureka Math* lesson videos.
Blogs • Implementation advice • Resources for teachers and parents	• Learn best practices for implementing *Eureka Math*. • Receive guidance on addressing various learning needs. • More effectively support at-home learners.
Virtual Facilitations • Multiyear sequence of professional development • Foundational sessions to prepare educators to implement the curriculum and customize it to meet student needs • Sustaining sessions to build educators' capacity and deepen educators' understanding of the curriculum	• Understand intentionality of learning design to target coherence in curriculum. • Learn to use powerful tools to support making the most of classroom learning. • Incorporate *Eureka Math* in customized lessons.

Appendix A: The Fundamentals of Effective Math Discourse

As introduced earlier in the section on Enhanced Learning Through Discourse, these six questions provide a synopsis of various frameworks for teachers to prepare for and facilitate effective math discourse. Here we elaborate on each question and ground it in established research.

Question 1: What knowledge is essential for student learning?

Identify the core learning outcomes.

"If we want students to ..., then we need to ...." This statement reinforces one of the practices of effective mathematics teaching—establish goals to focus learning (NCTM 2014). Establishing goals includes identifying clear learning outcomes, situating outcomes within coherent progressions, and using the outcomes to guide instructional decisions.

A clear understanding of the unifying ideas across lessons allows teachers to spend more time on fewer topics and focus deeply on the major work of each grade (NGA Center and CCSSO 2010). This focus helps students gain strong foundational understanding of concepts, develop fluency with skills and procedures, and build their capacity to apply key math ideas to solve problems inside and outside the classroom.

Select tasks that promote reasoning and problem solving about the core learning outcomes.

Part of preparing to facilitate any discussion involves selecting high-level tasks for students to discuss. These tasks should elicit thinking about the big ideas linking the lessons while also providing multiple ways to access and reason about the task (low floor, high ceiling). The mathematical nature of the task should invite all students to explain and justify their thinking, critique each other's ideas, use and connect multiple representations, look for structure and patterns, make conjectures, and form generalizations.

Question 2: How do we elicit and monitor student thinking about essential knowledge?

Elicit thinking about mathematical ideas.

It is important that every student has the opportunity to develop a strong positive identity as a learner of mathematics. This means that students need to be supported in engaging in grade-level content through multiple representations of math ideas and talking about their math thinking with diverse groups. They need multiple opportunities to make their math ideas stronger with deep understanding and flexible use of concepts and to make their math ideas clearer with more precise language and visuals (Zwiers et al. 2017).

Teachers can productively set the expectation that explanations of math ideas are an essential component of the sensemaking process. Using prompts such as "Can you tell us how you solved that?" or "Tell us about your strategy" lets students know that they are expected to *understand* the math, not just get answers. Follow-up questions help assess or advance student thinking (Smith and Bill 2008). Assessing questions are used to gather information about or probe thinking, prompting students to explain or clarify the steps they took to draw a model or solve a problem. Advancing questions are used to make the mathematical structures and connections visible or to encourage students to reflect on their work and justify its validity.

Amplify mathematical language.

Language plays a key role in students making sense of mathematics. Rather than trying to simplify or avoid the use of key mathematical language, teachers should make it more accessible by amplifying language (Walqui and van Lier 2010). *Amplifying language* means anticipating where students might need support in articulating their understanding in mathematical terms and providing multiple ways to access the concepts or terms. Organizing information clearly and coherently, using manipulative materials or visuals, and engaging in think-alouds are all possible ways to amplify content-specific language so students feel supported in their internalization of the math concepts.

Question 3: How do we engage students in each other's ideas?

Facilitate constructive conversations.

Conversations provide scaffolds for students to simultaneously make meaning of the key math ideas, communicate their understanding, and hear how other students express their understanding (Mercer and Howe 2012; Zwiers 2011). Effective discussions involve students in sharing experiences related to a task, posing and answering questions, and building common understandings. Throughout their discussions, students should be supported in using content-specific language to make and justify claims, communicate their reasoning, and critique the reasoning of their peers. Above all, students should be encouraged to make mistakes and provide each other with constructive feedback on ways to improve their use of strategies, models, and language in relation to key math ideas.

Foster student-to-student engagement.

Conversations can take place in many ways. Carpenter and colleagues (2015) categorize three levels of student engagement with each other's ideas:

- Comparing students' ideas
- Attending to the details of another's ideas
- Building on or adding to another's ideas

When students have a reason or purpose to talk and listen to each other, the interaction and engagement is more authentic. Teachers should, as often as possible, position student work as the focal point of a discussion. They might ask students to consider what is the same and different about the strategies or models used, what details about the work are efficient or inefficient, or what might be modified to improve the work. Using open-ended tasks that invite divergent thinking helps create situations where students want to share their ideas (which are not the same). Meaningful conversations depend on the teacher using student thinking to motivate and value efforts to communicate mathematically.

Question 4: How do we address errors or misconceptions?

Celebrate mistakes as opportunities to learn.

Mistakes and misunderstandings are natural parts of the learning process. However, making mistakes can often make students feel frustrated and discourage them from exerting the effort required to advance their understanding or skill. Teachers sometimes interpret a lack of student success, or "struggle," as a need to jump in and guide students through a task step by step. Instead, mistakes should be celebrated as we support students in their development of understanding and using mathematics content and language.

As teachers prepare instruction, they should work to anticipate the potential points of struggle in a lesson or common misconceptions about the key math ideas. Thinking about these in advance allows the teacher to prepare to support students in productive ways without removing the opportunities for reasoning, sensemaking, and constructive communication. One way to elevate and celebrate mistakes might be to (anonymously) display student work that contains an error or a misconception and then facilitate a discussion about where the error was made and how it might be fixed. Keeping errors anonymous focuses the attention on the thinking, not the author.

Support productive struggle as students work toward learning outcomes.

Whenever new learning begins, students should expect the details of the topic to be a bit fuzzy at first. Productive mathematics learners recognize that initial confusion will not last; clarity of concepts will come from sustained effort and appropriate supports. Instruction that embraces students' mistakes and misconceptions and leverages them as authentic learning opportunities leads to long-term achievement, with students more able to apply their learning to new or novel problems (Kapur 2010).

The types of tasks and questions we prepare and the feedback we provide send students messages about what we believe they can do. Providing feedback that values their efforts in persevering through a challenging task or their attempts at being more precise with their mathematical language, combined with specific insight into the strengths of their work and areas for improvement, develop students who are also more likely to take risks, face challenges head-on, and not give up at the first sign of difficulty.

Question 5: How do we connect and extend important math ideas?

Elicit connections among representations, methods, and ideas.

When students are consistently engaged in representing, discussing, and making connections among mathematical ideas in multiple forms, they demonstrate deeper understanding and greater ability to reason and problem solve (Fuson, Kalchman, and Bransford 2005). Visual representations, such as math drawings, diagrams, tables, or graphs, support discourse because they provide a record of student thinking that can be displayed, analyzed, discussed, and critiqued. Physical and visual supports also allow more students to access the grade-level content and contribute meaningfully to discussions (Fuson and Murata 2007).

Understanding is deepened when teachers pose tasks that promote the use of multiple representations or strategies and ask questions that elicit student thinking about the connections between the representations or strategies and the underlying mathematical ideas. This depth of understanding is related to the strength of connections between multiple representations, strategies, and broader math ideas, as well as students' perception of representations as tools to solve problems rather than as a learning outcome.

Extend student thinking through connections to new or more complex ideas.

Sometimes, students demonstrate that they clearly understand a strategy they are using and are starting to move toward more sophisticated strategies. Teachers can ask questions, provide tools, and suggest different representations to encourage students to think beyond what they have shared (Carpenter et al. 2015). This does not necessarily mean providing explicit instruction

about the use of a new tool, representation, or strategy, but rather simply nudging the student forward with the seed of an idea connected to what has already been produced. Extending student thinking requires that the teacher know how the current content fits within the broader progression of learning and what next step might make sense.

Extending student thinking can also support the development of fluency. The National Council of Teachers of Mathematics (NCTM 2000 and 2014) and the Common Core State Standards for Mathematics (CCSSM; NGA Center and CCSSO 2010) emphasize that procedural fluency follows and builds on a solid foundation of conceptual understanding, strategic reasoning, and problem solving. Fluency builds and extends from initial explorations and discussions, using and connecting informal strategies based on established meanings, and then extending to the use of general methods as tools for problem solving. Fluency develops over time with connections to foundational understanding and student-generated strategies. Rushing to fluency before understanding is solidified can undermine students' confidence, perpetuate the myth that being "good" at math means memorizing without connections to meaning, and cause math anxiety (Ashcraft 2002; Ramirez et al. 2013).

Question 6: How do we engage students in reflection about their learning?

Elicit reflection of progress toward academic, language, social, and emotional outcomes.

Using discourse to guide students toward learning outcomes requires that teachers attend to more than just whether answers are correct or not. Lesson preparation should include intentional moments to elicit responses from students, assess student understanding, and provide feedback to students about how they are progressing toward the desired outcome, along with opportunities for students to use feedback to reflect on their own progress and set their own goals for continual improvement (Heritage 2008).

Selecting and sequencing tasks and preparing key questions to illuminate common errors, invite the use of more efficient strategies, and draw out specific understandings throughout the lesson experience are approaches that can reveal student thinking and allow teachers to make in-the-moment decisions about next instructional steps. Student Debrief and Topic Discourse slide decks provide resources for teachers to "check in" on student thinking as they strive to advance the collective understanding, skill, and identity of the whole class.

Use evidence of student thinking to inform instructional decisions.

Decisions about next instructional steps should be based on established learning progressions and evidence of student learning. Sometimes, students may need additional scaffolds to support their access to the next lesson or topic suggested by curricular materials. Continually studying curricular materials to identify learning trajectories and anticipating student responses to tasks and questions, including common difficulties, mistakes, and misconceptions, can help teachers make decisions to better meet students' needs while advancing the collective understanding of the class.

Indicators of student progress should be based on the established learning outcomes and the connection to big math ideas rather than on whether student answers are correct. Generating correct answers or successfully solving problems does not always indicate understanding of the essential knowledge students are expected to learn. Effective teaching involves using the evidence gathered during a lesson to reflect on student progress resulting from instruction and to make decisions about the tasks, activities, questions, tools, and representations that will guide the direction of the next lesson. "Although there is no single best way to respond to student thinking, the response that the teacher gives should be intended to help students deepen their conceptual understanding while moving them forward toward procedural fluency and advanced mathematical reasoning" (NCTM 2014, 54).

Appendix B: Instructional Routines

Math Chat

Purpose: Creates an open-ended space for sharing mental math strategies and developing number sense, flexibility, efficiency, and accuracy

Grades: PK–12

Grouping: Whole class

Process	Considerations
1. Present a problem, number, or image. 2. Give students think time; student give a silent signal when ready. 3. Students share with partners. 4. Students share with the whole class.	• Pose a question that elicits thinking. • Listen actively as students share with partners. • Purposefully select and sequence ideas for class sharing that allow for rich discussion. • Identify specific vocabulary for students to use. • Focus on student thinking (verbal or written). • Ask questions to intentionally guide students to make connections and extend their thinking.

Always, Sometimes, Never

Purpose: Promotes sensemaking and mathematical discussion as students support a claim with examples and nonexamples

Grades: 1–12

Grouping: Small group

Process	Considerations
1. Present a statement. 2. Give students think time to decide whether the statement is always, sometimes, or never true. 3. Students share with a partner. 4. Students share with the whole class.	• Pose a statement that elicits thinking. • Listen actively as students share with partners. • Encourage the use of examples and nonexamples to support claims. • Conclude by coming to consensus. • Designate spaces in your room for students to move to, indicating their claim. • Provide tools (chart paper, markers, etc.) for students to record their thinking.

Co-Construction

Purpose: Provides structured support for contextualizing and decontextualizing problems, which help students build abstract reasoning

Grades: PK–12

Grouping: Partners (If students work individually, rename the routine Construction.)

Process	Considerations
Decontextualize 1. Present a context or situation. 2. Partners work to construct mathematical representations to match the context. 3. Pairs compare problems with other groups. 4. Students share with the whole class. **Contextualize** 1. Present a mathematical representation to the class. 2. Partners work to construct a context that could apply to the representation. 3. Pairs compare contexts with other groups. 4. Students share with the whole class.	• Pose contexts or math representations that elicit thinking. Math representations could be a model (e.g., number bond, tape diagram), a table or graph, or an expression or equation. • Encourage students to use labels and units as appropriate. • Focus class discussion on student responses.

Which One Doesn't Belong?

Purpose: Promotes metacognition and mathematical discourse as students use precise language to compare different representations or examples

Grades: PK–12

Grouping: Whole class

Process	Considerations
1. Present four images and invite students to study them. 2. Give students think time to decide which of the four images does not belong. 3. Students share with the whole class.	• Any of the four images could not belong; the focus is on constructing viable arguments. • Highlight responses that emphasize reasoning about key mathematics. • Ask questions to encourage precision of language, making connections, and asking questions of their own.

Numbered Heads

Purpose: Helps groups function effectively by encouraging students to build consensus and holds each student accountable for learning the material

Grades: 1–12

Grouping: Small group

Process	Considerations
1. Organize students into small groups and number each student 1 through X. 2. Pose a question. 3. Groups work together to answer the question. 4. Any group member could be the spokesperson. 5. Randomly select a number; students assigned that number share with the class.	• Use groups of three or four. • Pose open-ended questions to elicit divergent thinking.

Critique a Flawed Response

Purpose: Promotes effective communication techniques for critiquing others' work, correcting errors, and clarifying meaning

Grades: 3–12

Grouping: Whole class or small group

Process	Considerations
1. Present an incomplete or incorrect strategy or solution. 2. Give students think time to identify the error. 3. Students share with the class. 4. Give students time to solve the problem. 5. Students share suggestions for correcting the error.	• Purposefully select student work that allows for rich discussion about the error presented. • Encourage students to work the problem by using the strategy presented or to propose and defend a different strategy. • Conclude with consensus about how best to correct the error.

Take a Stand

Purpose: Supports students in making arguments and critiquing the reasoning of others

Grades: 1–12

Grouping: Whole class

Process	Considerations
1. Hang signs in the classroom. (See samples in the next column.) 2. Present a statement or problem; students stand near the sign that best describes their claim. 3. Give time for groups to discuss their reasoning. 4. Groups share to the class; students may change their minds. 5. Individuals reflect on key takeaways.	• Sample labels for signs: ◦ Strongly Agree, Strongly Disagree, Undecided ◦ One Solution, No Solution, Many Solutions ◦ Greater Than 100, Less Than 100, Undecided

Five Framing Questions

Purpose: Supports students in analyzing a work sample or solution strategy by providing them with questions to frame their thinking

Grades: PK–12

Grouping: Whole class or partners

Process	Considerations
Initial Analysis 1. Present student work to be analyzed (anonymous). 2. Notice and Wonder: What do you notice? What do you wonder? 3. Organize: What steps did the mathematician make? How do you know? **Advanced Analysis** 4. Reveal: Let's focus on [strategy/feature]. Where do you see that in this work? 5. Distill: What difference does [strategy/feature] make in this work? 6. Know: How is [strategy/feature] helpful to the [learning outcome/concept]?	**Partner Analysis** • Present the problem and give students time to solve. • Use the Five Framing Questions to invite partners to analyze each other's solution pathways. • Students share out their partner's work and connect it to the [learning outcome/concept]. **Support Language** • Provide sentence starters: ◦ I notice..., I wonder... ◦ The first thing you did was... I know because... ◦ I see how you... I can tell because... ◦ It is important to the work because... ◦ Looking at your work helps... because...

Stronger and Clearer Each Time

Purpose: Provides a structured, interactive opportunity for students to revise and refine their written language through rehearsal

Grades: 2–12

Grouping: Individual or partners

Process	Considerations
1. Present a problem with a claim or solution pathway. 2. Give students time to solve the problem independently and to write a justification for or against the proposed claim or solution pathway. 3. Partners exchange written explanations. 4. Partners ask clarifying questions and critique one another's responses. 5. Give students time to improve their explanations by applying the feedback given. 6. Partners exchange explanations and repeat the process.	• Provide specific vocabulary for students to use. • Direct students to give specific feedback about what is not convincing them about their partner's argument. • Circulate and listen as partners discuss. • Select student explanations that demonstrate improvement in clarity or precision to share with the class.

WORKS CITED

Ashcraft, M.H. 2002. "Math Anxiety: Personal, Educational, and Cognitive Consequences." *Current Directions in Psychological Science* 11, no. 5.

Carpenter, T.P., E. Fennema, M.L. Franke, L. Levi, and S.B. Empson. 2015. *Children's Mathematics: Cognitively Guided Instruction*. 2nd ed. Portsmouth, NH: Heinemann.

Fuson, K.C., M. Kalchman, and J.D. Bransford. 2005. "Mathematical Understanding: An Introduction." In *How Students Learn: History, Mathematics, and Science in the Classroom,* edited by M. Suzanne Donovan and John D. Bransford. Committee on How People Learn: A Targeted Report for Teachers, National Research Council. Washington, DC: National Academy Press.

Fuson, K.C., and A. Murata. 2007. "Integrating NRC Principles and the NCTM Process Standards to Form a Class Learning Path Model That Individualizes within Whole-Class Activities." *Journal of Mathematics Education* Leadership 10, no. 1. National Council of Supervisors of Mathematics.

Great Minds®. 2015. *Eureka Math®*. Washington, DC: Great Minds. https://greatminds.org/math/eurekamath.

Heritage, M. 2008. *Learning Progressions: Supporting Instruction and Formative Assessment.* Washington, DC: Council of Chief State School Officers.

Kapur, Manu. 2010. "Productive Failure in Mathematical Problem Solving." *Instructional Science* 38, no. 6.

Mercer, N. and C. Howe. 2012. "Explaining the Dialogic Process of Teaching and Learning: The Value and Potential of Sociocultural Theory." *Learning, Culture, and Social Interaction* 1, no. 1.

National Council of Teachers of Mathematics (NCTM). 1995. *Assessment Standards for Teaching Mathematics.* Reston, VA: NCTM.

NCTM. 2000. *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

NCTM. 2014. *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

National Governors Association Center for Best Practices and Council of Chief State School Officers (NGA Center and CCSSO). 2010. *Common Core State Standards for Mathematics.* Washington, DC: NGA Center and CCSSO. http://www.corestandards.org.

Ramirez, G., E.A. Gunderson, S.C. Levine, and S.L. Beilock. 2013. "Math Anxiety, Working Memory, and Math Achievement in Early Elementary School." *Journal of Cognition and Development* 14, no. 2.

Smith, M.S., V. Bill, and E.K. Hughes. August 2007. "Thinking Through a Lesson Protocol: A Key for Successfully Implementing High-Level Tasks." *Mathematics Teaching in the Middle School,* vol. 14. Reston, VA: NCTM.

Smith, M. and M.K. Stein. 2018. *5 Practices for Orchestrating Productive Math Discussions.* 2nd ed. Reston, VA: NCTM.

Walqui, A. and L. van Lier. 2010. *Scaffolding the Academic Success of Adolescent English Language Learners: A Pedagogy of Promise.* San Francisco: WestEd.

Zwiers, Jeff et al. 2017. *Principles for the Design of Mathematics Curricula: Promoting Language and Content Development.* Stanford Center for Assessment, Learning, and Equity (SCALE). Stanford University.

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