Why

Expressions and One-Step Equations

What new symbols and notation do students learn in this module?

With the introduction of variables and algebraic expressions and equations, grade 6 module 4 marks an important transition for students from the symbols they used in elementary grades, such as × and ÷, to the symbols that students use throughout the rest of their study of mathematics.

- In topic A lesson 2, students begin using the dot operator $\cdot$ for multiplication.
- In topic A lesson 3, students begin using exponential notation.
- In topic A lesson 5, students begin using a number next to parentheses to indicate multiplication, such as $3(4)$.
- In topic A lesson 5, students begin using the fraction bar as a grouping symbol and begin seeing operations in the numerator and/or the denominator, such as $\frac{20}{4+1}$.
- In topic B lesson 8, students begin using juxtaposition of a number and a variable or variables to indicate multiplication, such as $10x$ and $10xy$.

Beginning in topic B lesson 8, students recognize that there can be multiple ways of expressing terms that involve a fraction times a variable, such as $\frac{2}{3}y$ and $\frac{2y}{3}$, or $\frac{1}{2} \cdot 6w$, $\frac{1}{2}(6w)$, $\frac{6w}{2}$, and $3w$. Although very little work in grade 6 involves the product of two variables, it is mathematical convention to express terms with a single coefficient and to write the coefficient first followed by the variable factors, which are generally in alphabetical order. It is useful to model these conventions with students to lay a foundation for future work. However, accept any correct answer from students: If an answer is $3xy$, it is also correct to write it as $3yx$.

After the dot operator, juxtaposition, and parentheses are introduced to indicate multiplication, students rarely see or use the × symbol. The ÷ symbol is used in grade 6

Which expressions have the same value as $\frac{2}{3} \cdot 3^2 - 6 + 5$? Choose all that apply.

A. $\frac{2}{3}(3^2) - 6 + 5$

B. $\left(\frac{2}{3} \cdot 3^2\right) - (6 + 5)$

C. $\frac{2}{3} \cdot 3^2 - (6 + 5)$

D. $\left(\frac{2}{3} \cdot 3\right)^2 - 6 + 5$

E. $\left(\frac{2}{3}(3^2)\right) - 6 + 5$

Which expressions represent the product of $\frac{2}{3}$ and y? Choose all that apply.

A. $\frac{2}{3}y$

B. $\frac{2y}{3}$

C. $\frac{2y}{3y}$

D. $\frac{2}{3y}$

when dividing fractions to avoid complex fractions, such as $\frac{\frac{2}{3}}{\frac{3}{4}}$, which are introduced in grade 7. In grade 7 and beyond, students generally only use the fraction bar to indicate division.

This module does not ask students to simplify expressions. Why?

What does it mean to simplify an expression? Teachers often use the term *simplify* for different meanings. For example, teachers might simplify $3x + 5x$ by combining like terms to make $8x$, or teachers might simplify an expression like $2(3 + 4^2)$ by using the conventional order of operations. It is imprecise to use a single word, *simplify*, to indicate a variety of different mathematical procedures.

Because the meaning of *simplify* is unclear without further details, this module does not use that term in directions to students. Rather, the directions in problems in this module precisely state how the student is expected to manipulate a given expression to create an equivalent expression, such as *create an equivalent expression by combining like terms*, *factor the expression by using the greatest common factor and the distributive property*, and *write an equivalent expression with as few factors as possible*. Not only do these directions avoid vague terms such as *simplify*, they also reinforce the relevant concepts for students, such as the meanings of *equivalent expression*, *factor*, and *term*.

Students may encounter the term *simplify* in assessments and other resources. Use your best judgment for how and whether to introduce this term to students. Model precise vocabulary for students by using clear language whenever possible.

Should I teach my students to recognize mathematical keywords and the operations the keywords represent?

In general, no. A focus on translating keywords detracts from students' ability to comprehend and make sense of situations.

Two primary instances occur in this module in which students translate words, which many teachers think of as keywords, into algebraic expressions.

1. Translating phrases such as "3 less than a number x" into the algebraic expression $x - 3$: In this case, *less than* is not a keyword, it is a verbal description of a

Distribute and combine like terms.

$$2k + 3(4k + 5)$$

$$\begin{aligned} 2k + 3(4k + 5) &= 2k + 3(4k) + 3(5) \\ &= 2k + 12k + 15 \\ &= (2 + 12)k + 15 \\ &= 14k + 15 \end{aligned}$$

Factor the expression by using the greatest common factor and the distributive property.

a. $7x + 14$

$7(x + 2)$

b. $40g + 8h$

$8(5g + h)$

A	B
The sum of 3 and twice a number $3 + 2n$	$4 - 2.5g$ The difference of 4 and 2.5 times a number
C	**D**
One-half of the difference of a number and 12 $\frac{1}{2}(n - 12)$	$\frac{3}{4}(9x)$ Three-fourths of the product of 9 and a number

mathematical operation. When writing algebraic expressions to represent descriptions such as these, it is appropriate and valuable to teach students to recognize that *less than* indicates subtraction. In this example, "3 less than a number x" cannot be interpreted in any other way.

2. Translating real-world situations such as "Toby scores 3 points less than Scott scores" into algebraic expressions: In this case, *less than* does not necessarily indicate subtraction. It is critical that students have the ability to decontextualize this situation and recognize the relationships between the two quantities, rather than just assume that the words *less than* always indicate subtraction.

   For example, if the problem continues, "Toby scores t points. Write an expression to represent the number of points that Scott scores," then the appropriate algebraic expression is $t + 3$. In this situation, the phrase *less than* leads to an addition expression.

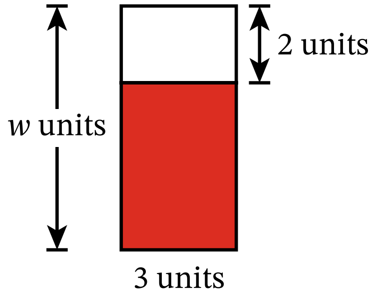
   Furthermore, a very similar-sounding sentence, "Toby scores 3 points, which is less than Scott's score, s," leads to the inequalities $3 < s$ or $s > 3$, which are very different from the intended expression. So the phrase *less than* can indicate different meanings in mathematics, depending on how it is used.

Because of examples like Toby's score, it is counterproductive to teach students to recognize keywords as exclusively representing a single operation, especially when students write expressions and equations from real-world contexts. Many of these keywords can indicate multiple different operations, depending on how they are used. Rather, focus on fostering students' ability to make sense of and decontextualize the situations in which these keywords appear.

Can students use substitution to prove that two algebraic expressions are equivalent?

Technically, no. Students cannot prove that two algebraic expressions are equivalent by using substitution. However, substitution is still a valuable tool for students to informally check whether two expressions are equivalent.

By definition, two algebraic expressions are equivalent if both expressions evaluate to the same number for every possible value of the variable. To prove that two algebraic



The rectangle is made up of two smaller rectangles.

b. Write two expressions that each represent the area of the shaded rectangle in square units.

$$3(w - 2)$$

$$3w - 6$$

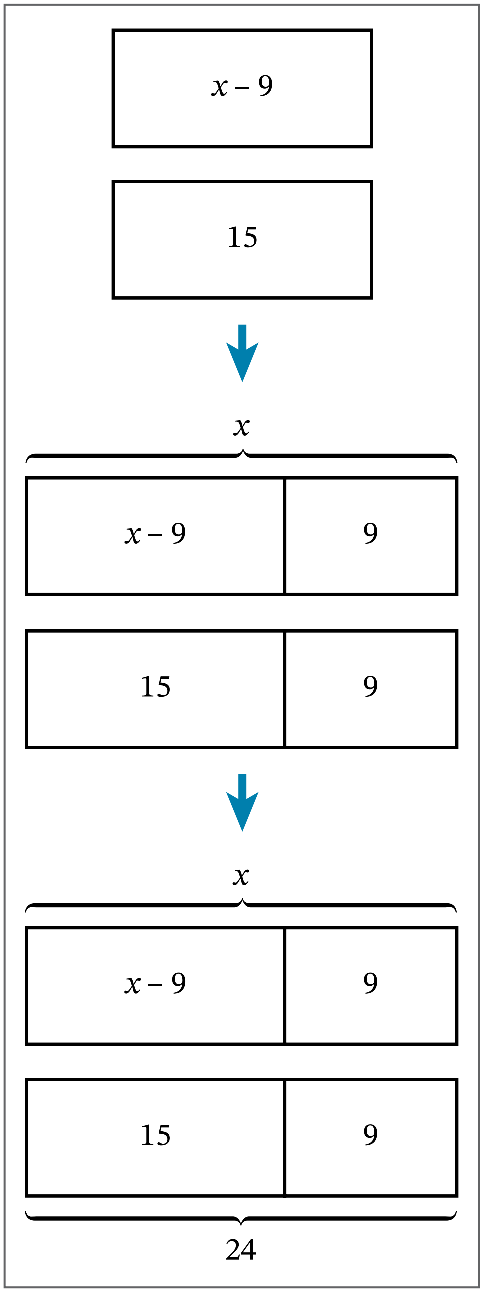
c. Use the distributive property to show that the expressions you wrote in part (a) are equivalent.

$$3(w - 2) = 3(w) - 3(2)$$
$$= 3w - 6$$

expressions are equivalent, students would have to evaluate the two expressions at every possible value of the variable, which is an impossible task. Instead, students discover throughout the module that applying a property of operation to an algebraic expression generates an equivalent expression.

However, substitution still serves a useful purpose. Students should practice substituting a few different values for each of the variables in a pair of expressions and evaluating the expressions. If the expressions evaluate to different numbers, then they are not equivalent. If the expressions evaluate to the same number every time, students at this stage of learning can be reasonably confident that the expressions are equivalent, even though substituting only a few values for each of the variables does not actually prove that they are equivalent. As students' learning progresses, they will develop a more sophisticated understanding of equivalent expressions.

Substitution and evaluation are referenced throughout topic B and topic C. In topic C lesson 12, students examine a situation in which a student uses substitution to test for equivalency and draws an incorrect conclusion, which helps build students' understanding of when and how to appropriately use substitution to informally check equivalency.

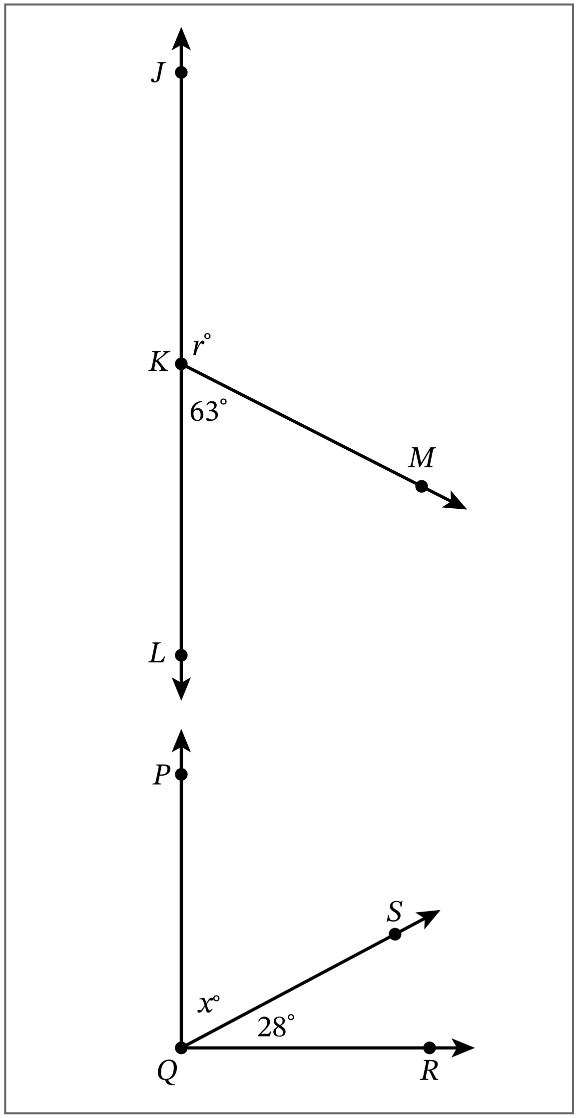


What types of equations do students work with in this module? What types of equations are beyond the scope of grade 6?

In topic D of this module, students learn to solve one-step, single-variable equations involving addition, subtraction, multiplication, and division, such as the equations $f + 8 = 17$, $x - 9 = 15$, $5f = 30$, and $\frac{x}{3} = 27$. These equations all feature a single operation with a variable, and they can be solved by using tape diagrams, arithmetic reasoning, or algebraic reasoning. At the end of the topic, students also encounter equations that may require an additional step, such as combining like terms on one side of the equal sign as in $0.62n = 82 + 65 + 39$.

$$0.62n = 82 + 65 + 39$$
$$0.62n = 186$$
$$0.62n \div 0.62 = 186 \div 0.62$$
$$n = 300$$

In grade 6, the values of the variables, coefficients, and constants in the equations that students solve are all nonnegative rational numbers. In addition, it is important that students are not presented with equations that have negative solutions, such as $x + 5 = 3$, or that require students to operate with integers, as these equations are beyond the scope of grade 6. In grade 7 and beyond, students solve equations that involve multiple operations with a variable, such as $2x + 5 = 3$, and that involve all types of rational numbers, including integers and negative rational numbers.



Why are students solving angle problems in this module?

Throughout module 4, you will notice problems involving expressions and equations that embed a variety of previously learned content, including ratios, rates, percents, greatest common factors, and geometry concepts such as area, perimeter, and angles. This design is intentional so that students have recurring practice of key concepts throughout the year.

After students are first introduced to angles and learn to draw, measure, and name them in grade 4, students do not formally work with angles again until they encounter supplementary, complementary, vertical, and adjacent angles in grade 7. Therefore, this module includes work with angles for several reasons.

- Angle relationships provide an ideal context for writing and solving one-step equations because students can write equations from a pair of angles that form a right angle or a straight angle.
- In grade 4, students solve addition and subtraction problems to find unknown angle measures on a diagram. Now that they have been formally introduced to variables and equations in grade 6, it is useful to revisit that past work so that students understand the connection between the arithmetic reasoning that they use in grade 4 and the algebraic reasoning that they use in grade 6.
- Pedagogically, reviewing some angle concepts in grade 6 ensures that students are prepared for grade-level work with angles in grade 7.