Why

Addition and Subtraction with Fractions

When should students put fractions in simplest form? Why?

Referring to the simplest form of anything can be tricky because what is simple to one person may not appear simple to another. Further, in some cases what is known as simplest form can be more difficult to interpret if a context is involved. For example, $\frac{54}{60}$ is a simpler form than $\frac{9}{10}$ if a problem asks how many hours someone works on a project.

Students are not expected to rename fractions in one form or another unless the context of a problem requires students to do so. For example, a number such as $\frac{30}{20}$ is an acceptable answer for a sum or difference. However, if $\frac{30}{20}$ is the result when a student is asked a question about how many more kilometers one person ran than another, students are expected to express their answer as a mixed number, such as $1\frac{10}{20}$. A mixed number is often easier to interpret than a fraction greater than 1 in context-based problems.

Why are students sometimes asked to express their answer in the largest unit instead of being asked to reduce the fraction or write their answer in lowest terms?

Much of the traditional language involved with fractions, such as *reduce* and *lowest terms*, may either cause confusion or be misunderstood. Let's first consider the term *reduce*. Reduce means to make something smaller in amount or size. It is inaccurate to say "reduce a fraction" because it is not the fraction that is reduced; rather the numerator and denominator are proportionally reduced to make a fraction with the same value. To avoid the misconception that reducing a fraction makes a fraction with a lesser value, students are not given the direction to reduce fractions.

Now consider the phrase *lowest terms*. The mathematical use of *term* is not introduced to students until grade 6, which is when students can make sense of a term as a single number, a variable, or a product of numbers and variables in an expression. In grade 4, students use *term*, but only when referring to any one item in an ordered list of items in a pattern. Because *lowest terms* is a phrase students cannot yet make sense of mathematically, students are not asked to write fractions in lowest terms.

Instead of being directed to reduce or write fractions in lowest terms, students are asked to express their answer by using the largest unit. This is a more accurate description because when a fraction is "reduced," it is actually written with a larger fractional unit. For example, the fraction $\frac{30}{100}$ is 30 hundredths in unit form. 10 hundredths has the same value as 1 tenth, so 30 hundredths has the same value as 3 tenths, and $\frac{30}{100}$ is equivalent to $\frac{3}{10}$. Tenths are a larger unit than hundredths, so the fraction $\frac{30}{100}$ written in the largest possible unit is the fraction $\frac{3}{10}$. Students enter grade 5 with a solid understanding of the term *unit.*

When students are finding equivalent fractions, why do they write the factors in the opposite order in grade 5 than in grade 4?

In grade 3, students interpret the factors in the expression 3×4 as 3 groups of 4. Therefore, in grade 4, students find equivalent fractions by writing an expression such as $\frac{3 \times 4}{3 \times 5}$. This is interpreted as tripling the number of units selected and tripling the total number of units, especially when using an area model to make an equivalent fraction. By grade 5, students develop a deeper understanding of multiplication and begin to move away from pictorial models and equal-groups thinking. They are comfortable enough working with the multiplication expressions that it is more natural to write the factors of the original fraction first (e.g., 4 and 5) followed by the factors they use to rename the fraction (e.g., 3 and 3), rather than vice versa. In grade 5, students continue to use the area model to find equivalent fractions but move toward renaming fractions numerically. For these reasons, equivalent fractions are presented with an expression such as $\frac{4 \times 3}{5 \times 3}$, which honors the knowledge students have gained over the years and allows them to attend to the needs of the problem instead of focusing on ordering factors in a particular way.